Engineering Solutions to Mechanics, Marine Structures and Infrastructures

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Review

Important Role of Theoretical Mechanics in Engineering

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Abstract: Theoretical mechanics, fundamental to engineering, is crucial for technological advancement in areas like mechanical design, energy, and aerospace. Despite its broad application, challenges arise when addressing complex systems or interdisciplinary studies, where current models and methods may fall short. The purpose of this paper is to synthesise examples of applications of theoretical mechanics in various engineering fields and to demonstrate its key role in solving practical problems. At the same time, the article will also discuss the current problems and challenges encountered in the application of theoretical mechanics, including the consideration of complex environmental factors in practical engineering and the difficulties brought about by the intersection of multiple disciplines. In order to address these challenges, the article will suggest future developments in theoretical mechanics, such as the development of more efficient computational methods and the introduction of innovative experimental techniques. These recommendations are intended to promote the further development of theoretical mechanics and to foster its wider application and deeper disciplinary cross-fertilisation in the field of engineering.

Keywords: Theoretical mechanics; Engineering research; Engineering application; Experimental technology; Interdisciplinarity

1. Introduction

As the landscape of disciplines such as mechanical engineering [1], civil engineering [2], energy engineering [3], and aerospace engineering [4] evolves and intertwines more deeply

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with everyday life, theoretical mechanics has risen to prominence at the heart of scientific inquiry. Some of the directions of application of theoretical mechanics in engineering are listed in Figure 1. Serving not only as a foundational pillar within the engineering domain but also as a catalyst for elevating these disciplines to unprecedented levels, theoretical mechanics is a relentless contributor to the welfare of humanity and the advancement of society. Within the realm of theoretical mechanics, the focus researches three principal areas: statics [5], dynamics [6], and kinematics [7]. It harnesses an array of sophisticated mathematical tools to conduct thorough derivations and computations, culminating in the formulation of precise, mathematically expressed conclusions. The swift evolution of science and technology has spurred the emergence of numerous sub-disciplines and interdisciplinary fields [8] within theoretical mechanics, introducing a richer tapestry of mechanical model representations [9] and significantly expanding the applicability of the field. Furthermore, the strides made in modern scientific and technological advancements, particularly the maturation of electronic computer technology [10], have fortified the study of theoretical mechanics. This has empowered researchers to tackle complex mechanical challenges that were previously intractable with conventional approaches, thereby enhancing the efficiency and precision of research endeavors. Looking to the future, as theoretical mechanics continues to expand and delve deeper into the engineering sector with the proliferation of new branches and interdisciplinary areas, we can anticipate a more extensive and profound impact. These burgeoning applications hold the promise of infusing perpetual vitality into human society and propelling it toward loftier echelons of progress.

Adopting a juxtaposed logical framework, this paper describes the three core research directions of theoretical mechanics and points out the pivotal position of theoretical mechanics as a basic discipline of mechanics in the field of engineering. Through an in-depth discussion of specific examples of the application of theoretical mechanics in a variety of fields, including mechanical engineering, civil engineering, energy engineering, and aerospace engineering, this paper makes a strong case for the pivotal role played by theoretical mechanics in the field of engineering. At the same time, combined with the analysis of examples, this paper also provides an in-depth analysis of the challenges and deficiencies faced by the application of theoretical mechanics in the field of engineering, which provides a valuable reference for future research and development. In the concluding part, this paper analyses the development trend of theoretical mechanics in the field of engineering, aiming at providing valuable references for researchers in related fields and promoting the further development and application of theoretical mechanics in the field of engineering.

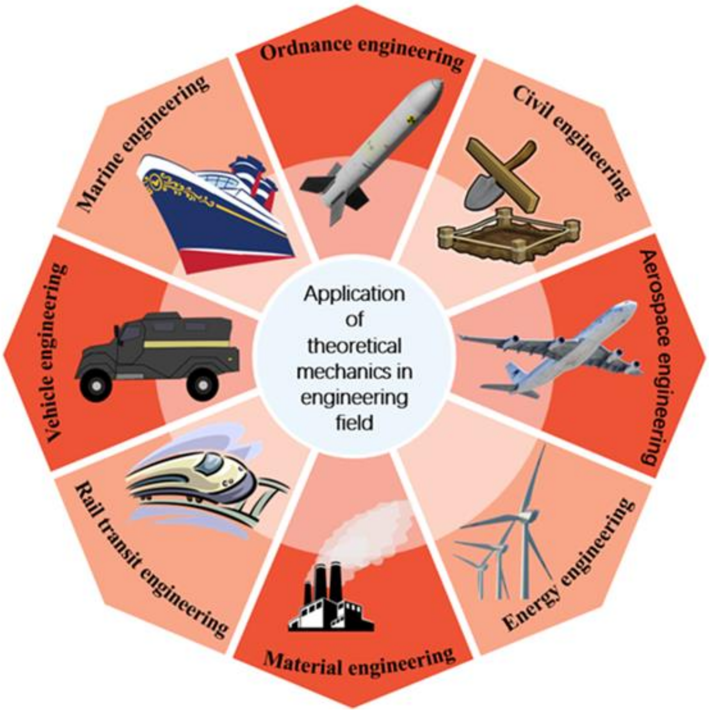

Figure. 1. Applications of theoretical mechanics in the field of engineering.

2. Concepts of Theoretical Mechanics and Scope of Inclusion

Theoretical mechanics, also known as classical mechanics, is the basis of the vast majority of engineering science and technology, the development of a total of three parts: statics, dynamics, kinematics. At the same time, key points in the development of theoretical mechanics are indicated in Figure 2.

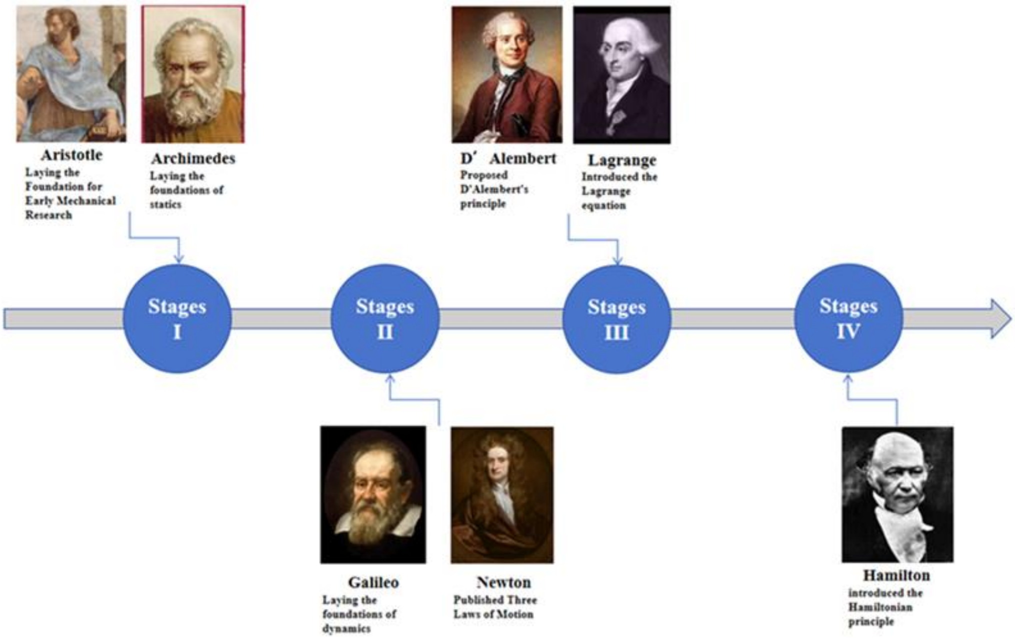

Figure. 2. Schematic representation of key points in the development of theoretical mechanics.

2.1 Statics

Statics, a key branch of theoretical mechanics, focuses on investigating the laws that govern the attainment of equilibrium by an object under the action of a system of forces. It has a long and distant history, dating back as far as the 3rd century BC, when Archimedes, a mathematical master in ancient Greece, put forward the famous lever balance formula and centre of gravity formula, laying a solid foundation for the establishment of statics. Into the Middle Ages, although Europe experienced a period of slow scientific development, scholars such as Al-Hazan continued to study the principles of mechanics in depth, planting the fire for subsequent developments. By the 18th century, with the rise of the scientific revolution, Euler and Lagrange, two giants of mathematics and physics, made epoch-making contributions to statics, and their work greatly enriched the theory of statics and pushed it to a new level. With the wave of the Industrial Revolution in the 19th century, the principles of statics began to be widely used in engineering and architecture [11] as an indispensable tool in the design and construction of all kinds of structures. Into the modern era, the development of computer technology and numerical methods, especially the introduction of numerical solution means such as finite element analysis [12], has made it feasible to solve complex static problems that were difficult to deal with in the past, and has greatly improved the accuracy and efficiency of engineering design. To this day, the principles of statics not only continue to play an important role in the traditional engineering field, but also with the emergence of new materials, new technologies, the scope of its application is also expanding, statics has become one of the cornerstones of modern engineering design and scientific research.

The three main research questions of statics:

1. Force analysis of an object: analyse the number of forces on an object and the point of action and direction of action of each force;
2. Simplification of force systems: Replacement of complex force systems with simple ones equivalently;
3. Equilibrium conditions for force systems: conditions for equivalence of a force system with a zero force system, and representation of the equilibrium conditions in the form of equations, and equilibrium equations for forces.

In the design of engineering structures, the principles of statics are used to determine data such as dimensional material and cross-sectional shape of members [13]; in mechanical engineering, static analysis is used to optimise the structural design of components to improve mechanical properties and service life [14]; in civil engineering, static analyses are used to ensure the safety and reliability of civil engineering structures [15]. Statics provides an important theoretical foundation for engineering design and scientific research.

2.2 Dynamics

Dynamics, as a core component of theoretical mechanics, focuses on revealing the intrinsic connection between the mechanical motion of an object and the forces it is subjected to. As early as in the ancient Greek period, philosophers put forward the preliminary concepts of "natural motion" and "forced motion" through careful observation of the motion of objects, providing a philosophical basis for the early exploration of dynamics. Into the 17th century, Galileo laid a solid foundation for the scientific study of dynamics through a series of precise experiments. Immediately after, Newton proposed the famous Three Laws of Motion in 1687, constructing a complete system of dynamics theory. Newton's second law, in particular, not only reveals the relationship between force and acceleration, but also derives the three major theorems of dynamics: the theorem of momentum, the theorem of momentum moment, and the theorem of kinetic energy. These theorems become powerful tools for dynamics modelling and kinematic characterisation. In addition, D'Alembert's principle, an important variant of Newton's second law, provides an alternative formulation of the fundamental laws of dynamics. The Dynamic-Static Method, established on the basis of D'Alembert's principle, is particularly suitable for solving practical engineering problems and is popular because of its practicality. With the birth of relativity and quantum mechanics at the beginning of the 20th century, the field of study of dynamics was greatly expanded. Einstein's theory of relativity, especially in describing the motion of objects in high-speed motion and in strong gravitational fields, made important corrections and additions to traditional dynamics [16]. The development of quantum mechanics, on the other hand, introduced the concept of dynamics into the microscopic world [17], providing a new theoretical framework for describing the motion of particles on the quantum scale. In contemporary times, the principles and methods of dynamics have been widely used in many engineering and technological fields such as aerospace, vehicle engineering, robotics, etc [18-20], and have become an indispensable part of modern science and technology. The study of dynamics not only advances engineering and technology, but also deepens our understanding of the laws of motion in nature.

The main research problem of dynamics:

1. Plasma dynamics: the study of the laws of motion of a single plasma point;
2. Dynamics of Plasma Systems: the study of the laws of motion of systems consisting of multiple plasmas;
3. Rigid body dynamics: the study of the laws of motion of rigid bodies.

In mechanical engineering, the dynamics theory is used to study the vibration characteristics, dynamical features and energy conversion processes of mechanical systems [21]; in civil engineering, kinetic theory is used to study the vibration characteristics and stability of structures, among others [22]; in aerospace engineering, dynamics theory is used to study the laws of motion, dynamic properties and structural strength of vehicles [23]; in vehicle engineering, dynamics theory is used to analyse vehicle handling, stability and braking performance [24]; in robotics engineering, dynamics theory can be used to develop control systems for robots [25].

2.3 Kinematics

Kinematics, an important branch of theoretical mechanics, focuses on the study of the evolution of physical quantities such as position, velocity, and acceleration of an object with respect to a specific reference system over time. The discipline focuses on exploring the laws of motion of objects from a purely geometric point of view, and does not deal with the physical properties of objects (e.g. mass) or the forces acting on them. In the study of kinematics, geometric methods are used to analyse and describe the state of motion of an object by establishing the equations of motion to accurately capture the trajectory of the object. Aristotle in the Ancient Greek period laid the foundations for the early development of kinematics with his initial classification and description of the motion of objects. Medieval scholars in the fields of mechanics and astronomy further laid a solid foundation stone for subsequent advances in kinematics. During the Renaissance, Galileo laid the scientific foundations of modern kinematics through his pioneering experimental studies of free-fall and parabolic motion. His work not only advanced the theory of kinematics, but also provided important insights into subsequent dynamics research. With the rise of the industrial revolution, the application of kinematics in engineering became increasingly widespread, especially playing a key role in mechanical design and system analysis [26-27]. Advances in computer technology have moreover made numerical methods increasingly important in kinematic analysis [28], greatly extending the ability of researchers to simulate and analyse complex kinematic behaviour. With the continuous emergence of new technologies, the research horizons of kinematics have further penetrated into the fields of nanotechnology [29] and quantum mechanics [30], entering a new era of exploring the laws of motion in the microcosm. This will not only deepen our understanding of the laws of motion in nature, but is also expected to bring revolutionary breakthroughs in the field of high technology.

Key research questions in kinematics:

1. Kinematics of points: the main study of the laws of motion of points in space.
2. Simple motions of rigid bodies: the main study of translation and fixed-axis rotation of rigid bodies in the plane or in space.
3. Plane motion of rigid bodies: the main study of rigid bodies in the plane of the law of motion.

In mechanical engineering, kinematics is used to analyse and design various mechanisms, such as linkage mechanisms, gear mechanisms, etc. Through kinematic analysis, the relative positions and velocities of the components in a mechanism can be determined, so as to optimise the design of the mechanism and improve its efficiency and reliability [31]; in biomechanics, kinematics can be used to study the human body's movement patterns, such as gait analysis, sports injury prevention, etc., which is of great significance for rehabilitation therapy, sports training, and the study of sports biomechanics [32]; in aerospace, kinematics is used to calculate parameters such as trajectory, velocity and acceleration of a vehicle to ensure flight safety and improve flight efficiency [33].

3. Core Formulations of Theoretical Mechanics and Its Subfields

3.1 Core Equations of Theoretical Mechanics

1. Components of force: force can be decomposed into components in any coordinate system:

$$\sum \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \tag{1}$$

2. Moment of inertia: the moment of inertia is a measure of an object's resistance to rotation:

$$I = \int r^2 dm \tag{2}$$

For an object of uniform density, the moment of inertia can be simplified to equation:

$$I = kMR^2 \tag{3}$$

Where k is the shape factor, M is the mass of the object, and R is the radius.

3. Parallel Axis Theorem: the rotational inertia can be calculated by the parallel axis theorem:

$$I = I_{cm} + Md^2 \tag{4}$$

Where I_{cm} is the moment of inertia of the object about the centre of mass and d is the distance from the centre of mass to the axis.

4. Centre of gravity: the centre of gravity of an object is the equilibrium point of the mass distribution of the object and can be calculated by integration:

$$\vec{G} = \frac{1}{M} \int \vec{r} dm \tag{5}$$

5. Lagrange's equations: the Lagrange's equations are a universal form in dynamics, applicable to both conservative and non-conservative force systems:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{6}$$

Where L is a Lagrangian quantity, q_i is a generalised coordinate, $\dot{q}_i$ is a generalised velocity, and Q_i is the generalised force.

6. D'Alembert's Principle: D'Alembert's Principle shows that the imaginary work done by a system in equilibrium is zero for any imaginary displacement:

$$\sum F \bullet \delta q = 0 \tag{7}$$

3.2 Core Equations of Analytical Mechanics

In analytical mechanics, one of the important concepts is Hamilton's principle, which is a set of fundamental equations that describe the dynamics of a system. Hamilton's equation:

$$\dot{q}_i = \frac{\partial H}{\partial q_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{8}$$

Where H is Hamiltonian measure, p_i is general momentum, q_i is general coordinate.

3.3 Core Equations of Fluid Mechanics

Fluid mechanics, as one of the important branches of mechanics, has three important equations. These equations can provide a reliable aid to the study of fluid mechanics.

1. Bernoulli's equation: bernoulli's equation, based on the principle of conservation of energy, describes the conversion and conservation of energy along the flow line in an inviscid, incompressible fluid. Bernoulli's equation shows that the total energy of a fluid is constant in the absence of work done by external forces during fluid flow. The expression of Bernoulli's equation is given below:

$$\rho\left(\frac{1}{2}\right)\rho v^2 + \rho g h = C \tag{9}$$

2. Navier-Stokes Equations: the Navier Stokes equations take into account the conservation of momentum of a fluid, including both volumetric (e.g., gravity) and surface (e.g., viscous) forces on the fluid. The general form of the equations is given below:

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \bullet \nabla \vec{v}\right) = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g} + \vec{F} \tag{10}$$

3. Continuity equation: the continuity equation is based on the principle of conservation of mass and describes the continuity of mass of a fluid during flow. For incompressible fluids, the continuity equation can be expressed as:

$$\nabla \bullet \vec{v} + \frac{\partial \rho}{\partial t} = 0 \tag{11}$$

For compressible fluids, the density varies in time and space, and the continuity equation needs to account for the change in density in the following form:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \vec{v} \right) = 0 \tag{12}$$

4. Applications of Theoretical Mechanics in Engineering

4.1 Application of Theoretical Mechanics in Energy Engineering

In the field of wind power generation, wind turbines are designed to accurately align the wind direction to optimise energy capture efficiency and to be able to adapt to variable climatic conditions [34-35]. At the same time, they must be equipped with self-protection mechanisms to withstand strong winds exceeding the rated wind speed to ensure the safety of the equipment. This is relatively easy to achieve for larger, technologically advanced wind turbines [36-37]. They are usually equipped with advanced computer-controlled systems that automatically adjust key components such as servo motors and hydraulic motors to achieve precise tracking and dynamic response to wind direction. However, for smaller wind turbines with lower costs, they are usually not equipped with such high-end control systems due to limited capital investment. In this case, the introduction of passive control systems is particularly necessary. Passive yaw control mechanisms, with their cost-effective and simplified design, are widely used in the power regulation and wind energy capture processes of small wind turbines [38]. This mechanism is capable of automatically adjusting the orientation of the wind turbine to changes in wind direction without external energy input, thereby increasing power generation efficiency while ensuring equipment safety.

In Narayana's study [39], he applied D'Alembert's principle to transform the dynamic system of a wind turbine into a static system for in-depth analysis. The schematic structure of a small turbine with D'Alembert force is shown in Figure 3. By this method he examined in detail the D'Alembert forces acting on the wind turbine, the generator, and the tail, and accordingly derived the exact equations describing these moments. In fact, the D'Alembert moment has a significant direct effect on the yaw dynamics of the wind turbine. Yaw dynamics [40] are a key factor in ensuring that the wind turbine is able to accurately align itself to the wind direction and activate the protection mechanism if the wind speed exceeds the rated value. Therefore, D'Alembert moments play a crucial role in the yaw control of wind turbines. They not only help to maintain the operational stability of the wind turbine, but also ensure that the wind turbine can respond accurately when the wind direction changes [41]. By calculating D'Alembert moments, Narayana developed a dynamic computer model of a small wind turbine system in MATLAB and simulated its yaw behaviour. The simulations indicate that at low rotational speeds, a small wind turbine system with yaw error can quickly reach its zero yaw error position.

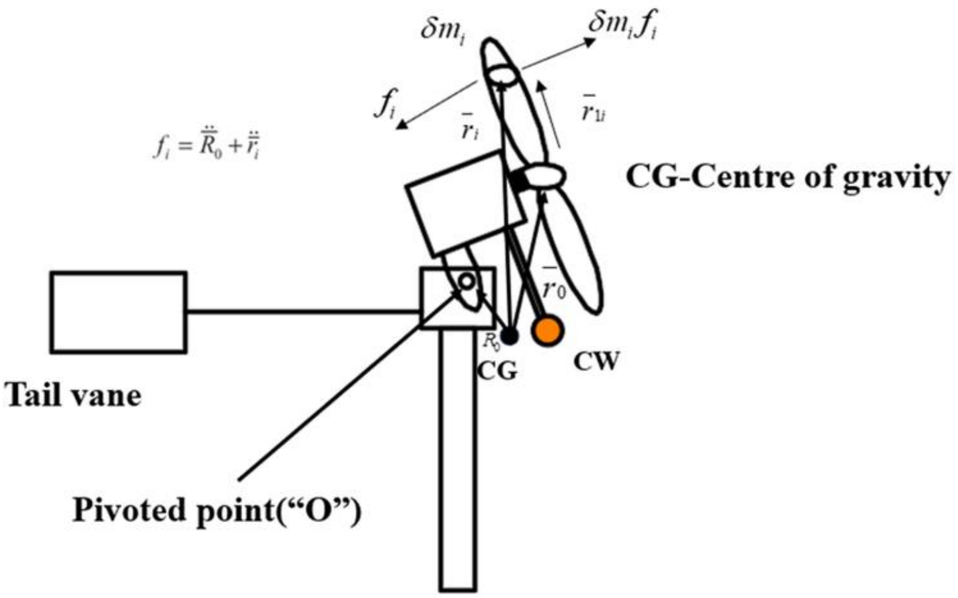

Figure. 3. Structure of small turbines with D'Alembert forces

In the above study, the moment of the D'Alembert force on "O" in the wind rotor, generator and counterweight system can be expressed as follows:

$$\begin{aligned} M_0 &= \sum\left(\bar{R}_0+\bar{r}_i\right)\times\delta\left(\ddot{\bar{R}}_0+\ddot{\bar{r}}_i\right) \\ &= \sum\delta m_i\left[\left(\bar{R}_0+\bar{r}_i\right)\times\ddot{\bar{R}}_0+\left(\bar{R}_0+\bar{r}_i\right)\times\ddot{\bar{r}}_i\right] \\ &= \sum\delta m_i\bar{R}_0\times\ddot{\bar{R}}_0+\underbrace{\ddot{\bar{R}}_0\sum\delta m_i\bar{r}_i}_{0}+\underbrace{\bar{R}_0\times\sum\delta m_i\ddot{\bar{r}}_i}_{0}+\sum\delta m_i r_i\times\ddot{\bar{r}}_i \end{aligned} \tag{13}$$

$$M_0 = m\bar{R}_0\times\ddot{\bar{R}}_0+\frac{d\left[\sum\left(\delta m_i\bar{r}_i\times\dot{\bar{r}}_i\right)\right]}{dt}$$

Thus, D'Alembert moments play a central role in analysing and modelling the yaw behaviour of wind turbines. It not only provides a key parameter for evaluating the performance of wind turbines in real-world operating environments, but also plays an important role in the design of more efficient and accurate yaw control systems. By accurately controlling the D'Alembert torque, the load on the wind turbine under high wind speed conditions can be significantly reduced [42], which effectively extends its service life and improves the overall operational efficiency and reliability. In addition, the application of D'Alembert's principle provides a solid theoretical support for the design and control of wind turbines [43], enabling engineers to more accurately predict and optimise the performance of wind turbines under a variety of wind speed and direction conditions. This theoretical foundation is of great significance in promoting the advancement of wind power generation technology, improving the efficiency of wind energy conversion, as well as promoting the wide application of renewable energy.

4.2 Application of Theoretical Mechanics in Civil Engineering

The tubular structural system [44], a major innovation in modern high-rise building design, provides a solution for building high-rise buildings that is both efficient and easy to construct. However, as building heights continue to rise, high-rise buildings inevitably face wind vibration problems [45]. The problem of vibration of buildings under the action of high-speed wind is becoming more and more prominent, and prolonged exposure to wind may lead to structural fatigue, damage to members, cracks or excessive deformation, and may even trigger structural collapse or serious damage. In addition, wind vibration-induced swaying and trembling of buildings can cause discomfort to occupants and affect their quality of life. Therefore, accurately calculating and evaluating the dynamic response of a building under the action of wind is a critical step in ensuring the safe and stable use of the building. During the design phase, the calibration of the collected data by means of approximate analyses [46] is essential to evaluate the performance of the different structural elements and their sizing options. This not only helps to optimise the structural design and improve the wind resistance of the building, but also provides a more comfortable and safer living environment for the occupants.

Based on an in-depth study of D'Alembert's principle, Mohsen-Maliknejad proposed an approximate formulation for predicting the structural dynamic response of tubular high-rise buildings [47]. His research innovatively considered discrete tubular structures as an equivalent collection of orthotropic anisotropic plates [48] and delved into framed tube structures [49] as a continuum. By applying the compatibility condition for deformation and combining it with D'Alembert's principle, he derived the governing dynamic equations for tubular structures and their corresponding natural boundary conditions. Further, Malik derived the characteristic equations and boundary conditions by assuming that the tubular structure undergoes simple harmonic free vibration [50] and expressing the transverse displacements in the form of separable variables. Using the Rayleigh-Ritz method [51], he solved the ordinary and nontrivial solutions of these equations to provide closed form analytical solutions for calculating the self-oscillation frequencies and corresponding vibration modes of high-rise silo structures. In order to verify the effectiveness of the proposed method, Maliknejad selected two typical tubular high-rise buildings as case studies. By comparing the results of theoretical calculations with actual observations, he fully demonstrated the advantages of the method in terms of accuracy, simplicity and reliability. By comparing the results of theoretical calculations with actual observations, he fully demonstrated the advantages of the method in terms of accuracy, simplicity and reliability.

In that study, D'Alembert's principle was used not only to verify the deformation compatibility between the structures [52], but also to derive the control conditions, thus ensuring the accuracy of the analysis process. Overall, the application of D'Alembert's principle makes the dynamic response analysis of tubular structures more feasible, transforming an otherwise complex vibration problem into a solvable system of linear differential equations. This shift has greatly simplified the analysis process, allowing researchers to efficiently calculate the natural frequencies and mode shapes of structures. This method is particularly suitable for the preliminary design phase, which usually requires iterative analysis and comparison of numerous options with widely varying structural characteristics [53]. By applying the D'Alembert principle, not only is the workload of the designer substantially reduced, but design efficiency is increased. This application fully demonstrates the great application potential of theoretical mechanics in the field of civil engineering, and provides new directions and possibilities for future progress and innovation in civil engineering technology.

4.3 Application of Theoretical Mechanics in Materials Engineering

In the field of materials science, the use of D'Alembert's principle or Hamilton's principle to derive the governing equations for the object of study is quite a crucial aspect. These governing equations provide researchers with a reliable tool for accurately describing the behaviour of materials under variable external conditions, this includes mechanical, thermal, electrical and magnetic behaviour under mechanical loading, temperature change, electric field action or magnetic field influence [54]. Through these equations, researchers are able to predict and model the response of materials in specific environments, thus providing a theoretical basis for material design and performance optimisation [55]. The application of the governing equations has enabled researchers to design new materials with specific performance requirements [56] or to systematically improve existing materials for specific industrial applications. In some studies, in addition to theoretical derivation, researchers need to construct simulation models to evaluate the performance of materials under actual working conditions through computer simulation, which provides an important reference for experimental testing and industrial application of materials.

In Reza Kolahchi's study [57], he combined nonlocal effect [58] and viscoelastic properties [59] to perform a detailed dynamic analysis and stability study of laminated composites and sandwich structures [60]. In this process, D'Alembert's principle played a key role in enabling the researchers to derive the governing equations of motion to accurately describe the behaviour of sandwich nanoplates under dynamic loading. By applying D'Alembert's principle, Kolahchi developed a comprehensive energy balance equation that takes into account the strain energy of the system, the work done by inertial forces, and the work done on the system by external loads. Similarly, in the study of Wenliang Gao [61], this Chinese scholar derived the governing equations describing the wave propagation behaviour in a metal foam plate based on Hamilton's principle and combined with different plate theories [62]. His study not only covers the theoretical derivation, but further evaluates the effects of porosity, GPL distributions, porosity coefficients, GPL volume fraction and geometry on the wave propagation dispersion relation, providing insights into the propagation properties of waves in metal foam sheets.

D'Alembert's principle and Hamilton's principle, as the two pillars of theoretical mechanics, play a crucial role in materials science research. Theoretical mechanics provides not only a fundamental framework for understanding and predicting material behaviour, but also a powerful set of analytical tools. Through the application of theoretical mechanics, researchers are able to reveal the intrinsic connection between the micro structure and the macroscopic mechanical properties of materials [63-64], and to study in depth the stress distribution [65], performance evolution [66] and other key issues of materials under different environmental conditions. Based on the analysis of mechanical models, researchers are able to accurately design and systematically optimise materials to meet specific engineering needs. Overall, theoretical mechanics plays an irreplaceable role in materials science research. It not only deepens scientists' understanding of the mechanical behaviour of materials, but also offers the possibility of predicting and controlling their performance in diverse application scenarios. The use of theoretical mechanics promotes the development of new materials, but also promotes the continuous improvement of the performance of existing materials, lays a solid theoretical foundation for the progress of materials science and opens up broad prospects for development.

4.4 Application of Theoretical Mechanics to Aerospace Engineering

The importance of aerospace engineering to modern society cannot be overstated. Its importance lies not only in the promotion of technological progress and economic growth, but also in its key role in enhancing the quality of life of the population, promoting scientific exploration and safeguarding national security. With the continuous progress of science and technology, flexible structures [67] have been widely used in modern spacecraft design due to their significant advantages such as light weight and low energy consumption [68-69]. These designs not only enhance the functionality of spacecraft but also effectively reduce launch costs. However, the application of flexible structures on spacecraft also brings new challenges. Poor coupling between transverse defects and attitude dynamics may significantly affect spacecraft performance. This coupling may lead to structural vibrations that affect the stability and control accuracy of the spacecraft, thus posing a threat to the success of space missions. Therefore, how to effectively control the attitude and vibration of flexure spacecraft has become one of the key topics of current research in aerospace engineering.

Facing how to effectively control the spacecraft's spatial flexible attitude and vibration when it encounters complex situations such as unknown actuator failures and input disturbances, Professor Liu Zhijie proposed an innovative solution [70]. He developed a novel adaptive fault-tolerant control strategy [71] and a corresponding adaptive controller design methodology [72]. The control strategy specifically addresses the problem of how to effectively suppress the vibration generated by the flexible plate structure during attitude stabilization process. By adaptively adjusting the control parameters, it is possible to achieve effective suppression of flexible plate vibration without relying on an accurate model of the system, thus safeguarding the critical components of the spacecraft from damage. The adaptive controller design method, on the other hand, is able to maintain spacecraft attitude and vibration stability in extreme cases of unknown input disturbances and actuator failures. This design takes into account the robustness of the spacecraft control system [73] and ensures that the attitude and vibration of the spacecraft are under control even when the system suffers from unexpected failures or external disturbances.

By combining the kinetic and potential energies of the system, as well as the variation of the virtual work, the researchers derived control equations describing the motion of a flexing spacecraft. Meanwhile, using Hamilton's principle, he developed a model of a complex system integrating partial differential equations and ordinary differential equations, which accurately describes the coupling effect between the rigid body attitude and the vibration of the flexible plate [74]. Theoretical mechanics, which underpins the entire study, provides the researcher with a key tool for understanding and analysing the motion of a flexing spacecraft in three dimensions, covering all aspects from rigid body dynamics to flexible body dynamics. The importance of theoretical mechanics in aerospace engineering cannot be underestimated. It not only provides solid theoretical support for the design and analysis of aeronautical and aerospace systems, but also plays an indispensable role in optimising the performance of the vehicle [75], ensuring the successful completion of the mission, enhancing the effectiveness and safety of the vehicle [76], reducing the cost of research and development and manufacturing, and playing an indispensable role in advancing the entire field of technology.

4.5 Application of Theoretical Mechanics in Railway Engineering

As an efficient, environmentally friendly and convenient mode of transport, rail transport has penetrated into people's daily lives in today's society and plays an important role in continuously improving people's quality of life and sense of well-being. China has a world-leading high-speed railway system, and as of 2020, the operating mileage of high-speed railways in the country has reached 35,000 kilometres, making it an important part of the country's transport infrastructure. Every year, China's railway system carries a huge volume of passengers and goods in circulation, especially during peak times like the Spring Festival, when the railway becomes the preferred mode of transport for hundreds of millions of people returning to their hometowns and journeys home. Ensuring the safety of railway transport is therefore a crucial task for railway workers and related researchers. In the sophisticated and complex operation system of high-speed railway, the bridge embankment transition section is often a key weak link [77], and its stability and safety are crucial for the reliable operation of the whole railway line. This particular region is subjected to dynamic loads generated by trains passing at high speeds [78], which, together with possible multiple effects such as environmental factors and material ageing, make it more susceptible to performance degradation and structural damage compared to other parts [79-80]. Therefore, careful study and reinforcement of the bridge embankment transition section is of great significance in preventing potential safety risks and enhancing the durability and overall performance of high-speed railways. Through targeted design optimisation and maintenance strategies, the bearing capacity and deformation resistance of this critical area can be effectively enhanced, thus providing a solid guarantee for the long-term stable operation of high-speed railways.

Hu Ping hopes to determine the dynamic response of the bridge embankment transition section by studying the coupled train-track-roadbed system. He used numerical simulation techniques to circumvent the limitations of field investigations and tests in order to study in depth the dynamic behavioural characteristics of transition sections under different operating conditions [81]. Based on the weak energy variational principle, D'Alembert's principle, as well as the overall Lagrangian format and the multi-rigid body dynamics theory [82], Hu Ping's study constructed a coupled system model of train-track-roadbed interaction at the numerical level. With this model, he analysed in detail the specific effects of track geometries such as track curvature and differential settlement of the roadbed on the dynamic performance of the bridge embankment transition section. The findings reveal that differential settlement and track surface bending are the key factors triggering the irregularities in the dynamic response of the bridge embankment transition section. Based on these findings, Hu Ping further proposed a targeted solution strategy to optimise the design and maintenance of bridge embankment transition sections to enhance the stability and safety of high-speed railways. In the study, the researcher constructed the dynamic equations of the coupled train-bridge-roadbed system using D'Alembert's principle. By taking inertial forces into account, the researchers simplified the analysis process of the dynamic system by transforming the complex dynamics problem into a more manageable statics problem. In addition, he succeeded in deriving the governing equations of the system, which profoundly describe the mechanism of interaction between the train, the track and the roadbed. Theoretical mechanics provides researchers with a solid theoretical framework covering a number of key areas such as rigid body dynamics, elastic mechanics [83] and vibration theory [84]. These theories are not only essential for understanding and modelling the dynamic behaviour of coupled systems, but are also indispensable for analysing and interpreting experimental measurements, validating the accuracy of numerical models, and assessing the validity of models. More importantly, the application of theoretical mechanics is not only limited to basic research, it also widely penetrates into all aspects of rail traffic engineering. Theoretical mechanics plays a crucial role in the design of bridges, tunnels, and track structures, in the analysis of dynamics, and in the assessment of safety [85].In conclusion, theoretical mechanics provides a solid theoretical foundation for the design, analysis, optimisation and maintenance of rail engineering. It not only ensures the efficient, safe and reliable operation of the rail transport system, but also promotes the continuous development and innovation of the rail transport technology, and lays a solid stepping stone for the exploration and progress of the future rail transport.

4.6 Application of Theoretical Mechanics in Ship Engineering

Ships play a pivotal role as a key tool for global trade and transport [86]. They are tasked with transporting all types of bulk goods, which include but are not limited to raw materials, commodities, and energy resources. For those countries that border the sea and rely on marine resources for trade and economic development, ship engineering is naturally an important driver of national economic growth. At the same time, in order to safeguard maritime rights and interests and territorial security, the need for military vessels is equally urgent for countries bordering the sea. Ship engineering plays a crucial role in ensuring that these ships have excellent performance and strong combat capabilities. It is not only about the country's military defence capability, but also a reflection of the country's comprehensive strength. As such, marine engineering has a profound impact on fuelling global economic prosperity and promoting social development. It not only supports the smooth conduct of international trade, but also has an indispensable role in national security and global strategic balance.

In ship engineering, theoretical mechanics can be used to analyse the forces on a ship during navigation [87] to ensure the safe navigation of the ship. Rafael Bardera used an innovative approach in his research [88] to accurately determine the forces and moments acting on the hull of a ship by performing careful balance measurements on a small-scale ship model. This approach not only improves the accuracy of the measurements, but also deepens the understanding of the ship's force mechanisms. In addition, the researchers used low-speed wind tunnel experiments to obtain information on the aerodynamic loads acting on the ship's hull. These valuable data provide a scientific basis for calculating the power required for ship motion and also point the way to improving the design of ship control systems. Through the optimisation of the ship control system, the navigation stability of the ship can be significantly improved, and its adaptability and reliability in the complex marine environment can be enhanced.

The principles of theoretical mechanics are used to design experiments to determine the aerodynamic forces and moments acting on small scale ship models [89,90]. Through these experiments, the researchers were able to accurately capture the aerodynamic effects acting on the hull of the ship, providing key data for subsequent ship design and performance optimisation. In addition, theoretical mechanics has been applied to analyse the stability of ships in depth and to assess the stability of full-scale ships [91]. Such analyses help engineers to predict and improve the performance of ships in real marine environments, ensuring their robustness in the face of complex sea conditions [92]. Theoretical mechanics also plays an important role in analysing the dynamic response of ships, which provides more effective control strategies for the design of ship control systems. The implementation of these strategies can significantly improve ship manoeuvrability and navigational safety [93]. As a cornerstone in the development of marine engineering, theoretical mechanics is also involved in the critical tasks of determining the buoyancy and equilibrium conditions of a ship, carrying out kinetic analyses of the motion of a ship in waves, and calculating the loads imposed on a ship by waves [94]. These efforts are not only critical to current ship design and construction technologies, but also drive continued progress and innovative development of related technologies.

4.7 Application of Theoretical Mechanics in Vehicle Engineering

With the acceleration of industrialisation, automobiles have been deeply integrated into people's daily life and work, becoming an indispensable means of transport in modern society. They provide great convenience for people's travelling and also greatly improve the efficiency of cargo transportation. However, this has been accompanied by a significant increase in car ownership per capita, a phenomenon that has given rise to a number of environmental problems, such as serious air pollution and increasing energy consumption. Therefore, it is particularly crucial to fine-tune the optimisation of the vehicle's form design and its components [95].This can not only significantly reduce the energy consumption of the car in the operation process, but also effectively reduce tailpipe emissions, and play a positive role in alleviating environmental pollution. By adopting advanced aerodynamic design, lightweight materials, and improved powertrains, it is possible to improve the fuel efficiency of automobiles to a certain extent [96], as well as reduce their negative impact on the environment.

In a study for SUVs [97], the researchers paid special attention to the effect of the shape of the stern on the aerodynamic drag of SUVs. Given that SUVs tend to be accompanied by higher CO2 emissions and energy consumption compared to medium-sized vehicles due to their unique body design and larger vehicle mass, careful design and optimisation of the SUV vehicle shape to reduce driving resistance is an effective way to improve energy efficiency. In this study, the researcher conducted an in-depth analysis of the aerodynamic drag forces experienced by SUVs during driving [98] based on the basic principles of theoretical mechanics. Using the dynamics equations, they simulated the dynamic behaviour of the SUV under different vehicle speeds, accelerations and force states. The researchers used morphing technique to accurately model the shape of the stern of the SUV and applied deformation and stress analysis methods from theoretical mechanics to systematically optimise the aerodynamic performance of the vehicle. In addition, the researcher has carried out an exhaustive theoretical analysis of the air resistance of the car by using the calculation method of theoretical mechanics on pressure distribution. Through these methods, researchers are able to identify and quantify the key factors affecting the aerodynamic performance of SUVs and propose design improvements accordingly.

In addition to its application in the aerodynamic design of automobiles, theoretical mechanics is an important tool for the structural analysis of automotive components. It helps engineers to study the loads that key components of a vehicle can withstand under normal conditions of use as well as under extreme operating conditions. When designing the powertrain of an automobile, the principles of theoretical mechanics are used to analyse in depth the load characteristics and transmission efficiency of the drive shafts, gears, and other important components [99], to ensure that these components are able to operate stably and reliably under a variety of operating conditions. In addition, theoretical mechanics is also used to investigate the vibration phenomena generated during vehicle travelling. By analysing the root causes and effects of vibration, engineers can design more refined suspension systems and vibration isolation devices to effectively control and mitigate vibration, enhancing ride comfort and vehicle smoothness [100]. Overall, the widespread use of theoretical mechanics has provided a powerful set of tools and methods for the development and study of vehicle engineering. It not only ensures vehicle stability, handling and safety, but also provides a theoretical basis for performance optimisation and innovative design. Looking to the future, with the continuous integration of theoretical mechanics with new materials and technologies, it will continue to promote the progress of vehicle engineering and inject new vitality into the sustainable development of society. Through the in-depth research and application of theoretical mechanics, future cars will be more efficient, environmentally friendly and intelligent, bringing more convenience to human life.

4.8 Application of Theoretical Mechanics in Weapons Engineering

As Dulay emphasises in his book Air Power, mastery of air power can significantly influence the course and ultimate outcome of a war. Therefore, the role of close-range ground air defence units in warfare should not be underestimated, as they not only provide effective countermeasures against enemy low-altitude vehicles, but also provide valuable support to their own air units [101]. Currently, short-range ground-based air defence systems are mostly deployed on armour and delivery vehicles, which are designed to give them excellent mobility to respond quickly to changing battlefield requirements [102]. An in-depth study of the dynamic characteristics of these mobile launch units when firing missiles is of great significance in optimising the existing control systems and upgrading the operators' practical skills. By accurately analysing the dynamics of the missile launch process, engineers and tactical specialists can design more efficient and responsive air defence systems, thereby improving overall battlefield defence capabilities. In addition, a careful study of the dynamic performance of mobile launch units can help military commanders better understand the tactical applications in different battlefield environments and develop more accurate and effective operational plans.

In the study, the researcher used the "semi-vehicle model" approach [103] to analyse the planar motion of the system in order to determine the characteristic laws of motion of the missile launcher power system [102]. In order to derive the mathematical model of the system, the researcher borrowed the second-order Lagrange differential equations:

$$\frac{d}{dt}\left(\frac{\delta E_k}{\delta \dot{q}}\right)-\frac{\delta E_k}{\delta q}+\frac{\delta E_p}{\delta q}+\frac{\delta D_{isp}}{\delta \dot{q}}=F\ , \tag{14}$$

Where E_k is kinetic energy, E_p is potential energy, D_{isp} is damping energy, F is a generalised force vector, q is a generalised coordinate, and $\dot{q}$ is a generalised velocity. The numerical model of the system consists of a total of five second-order differential equations, including expressions for kinetic, potential, and damping energies, as well as five degrees of freedom and five dependent variables of the system.

The model accurately captured the dynamic response characteristics of the ultra-short-range mobile launch unit during the launch process [104]. The model is shown in Figure 4. Subsequently, the researcher used MATLAB software with advanced numerical methods to simulate the whole system. The data obtained from the simulations provide a comprehensive picture of the changes in dynamic parameters such as displacement, angular displacement, velocity, angular velocity, acceleration, and angular acceleration of the key components of the launch system at various launch angle settings. With the help of the constructed mathematical model and the data generated by the simulation, the researchers were able to accurately reconstruct the angular motion dominated by the transmitter in the training simulator as well as the linear motion of the truck body. Accurate mastery of these motion laws and parameters is crucial for the development of simulation systems for ultra-short-range mobile launch units, and they not only help to improve the fidelity of simulation training, but also provide a theoretical basis and data support for the design of control systems for the launch units [105] and the optimisation of the operation process. In addition, these research results are of significant significance for improving the launch accuracy and response speed of ultra-short-range mobile launch units, and enhancing their adaptability and reliability in the changing battlefield environment. As the development of the simulation system continues, it will be able to provide operators with a training environment that is closer to the real world in the future, thus further improving the combat capability and reaction speed of the troops.

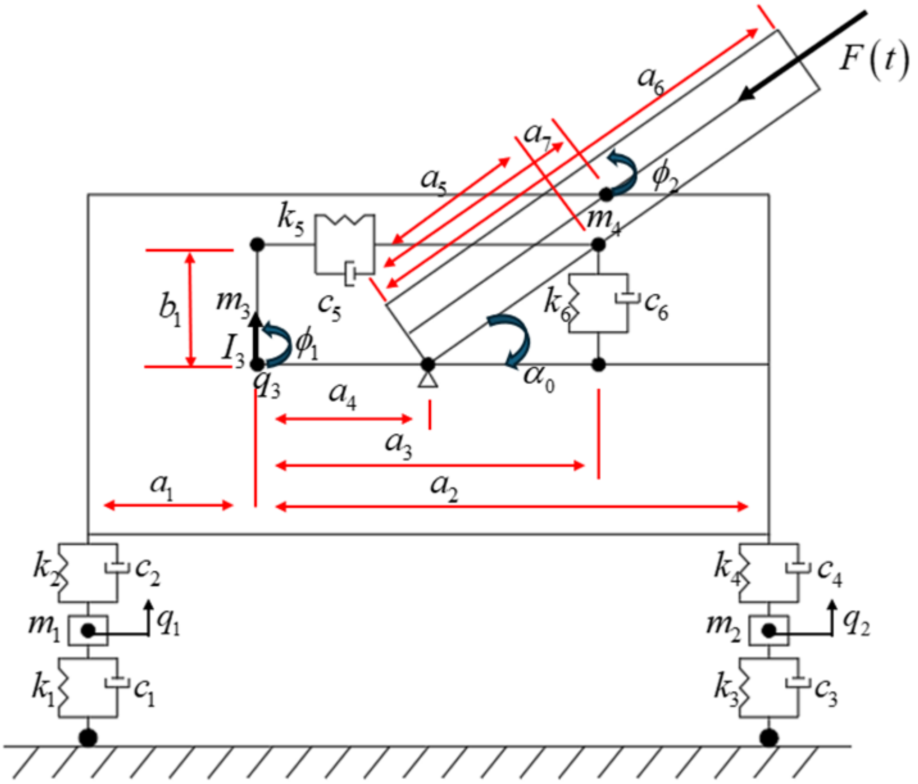

Figure. 4. Schematic of the dynamic model for ultra-short-range MFUs

In the field of weapons engineering, theoretical mechanics not only provides a solid theoretical foundation for key aspects such as weapon system design, control system development and ballistic calculations [106], but also greatly promotes the development of weapons engineering by combining with the principles of other disciplines. Among them, ballistics, as a core branch in weapons engineering, focuses on the kinetic analysis of the entire motion of a projectile from launching to hitting the target [107]. The principles of theoretical mechanics enable researchers to accurately track and predict the flight trajectories of projectiles, including the aerial trajectories of missiles and the underwater trajectories of torpedoes [108], which provides an important support for improving the accuracy and reliability of weapon systems. In addition, the combination of theoretical mechanics and aerodynamics allows for a more in-depth analysis of the effect of air resistance on the trajectory of the projectile [109]. This interdisciplinary fusion analysis not only enriches the research content of ballistics, but also provides a scientific basis for the optimisation of the performance of weapon systems under different environmental conditions. To sum up, theoretical mechanics is extremely widely used in weapon engineering, which not only provides a reliable foundation for theoretical research, but also injects a constant flow of innovative power into the design, improvement and development of weapon systems by combining with other disciplines. Through in-depth research and application of theoretical mechanics, weapons engineering can continuously break through technical bottlenecks and design more efficient and accurate weapon systems to meet the current complex needs.

5. Challenges and Limitations of Theoretical Mechanics in Engineering Applications

Theoretical mechanics plays an indispensable role in many engineering fields, not only as a cornerstone for analysing and designing engineering systems, but also as a core for building theoretical frameworks. In real-world engineering practice, since system design often involves multiple physical fields and interactions, it is challenging to build a mechanics model that fully accounts for all the complexities.

In the design of high-speed trains and aircraft, for example, engineers must take into account aerodynamic effects such as excitations, boundary layer separations, and other phenomena [110-112], which have a significant impact on the performance of the vehicle. While dealing with these design problems, theoretical mechanics models need to simplify the nonlinear effects in a reasonable way to ensure the practicality of the model, and this may also have some impact on the accuracy of the model. Similar challenges exist in the design and analysis of mechanical systems. For example, complex stress distributions can arise in regions where gears mesh or bearings are in contact [113]. When modelling mechanics, non-linear factors such as friction and contact stiffness must be taken into account, which have a direct impact on the dynamic behaviour and load carrying capacity of the system [114].

The construction of theoretical models is a scientifically rigorous process, which often relies on accurate experimental data as support. The accuracy of experimental data plays a decisive role in the predictive ability of the theoretical model, and any deviation may lead to deviation in the predictive results of the model. In practical engineering applications, due to the complexity and variability of the environment and operating conditions, it may be difficult for the theoretical model to fully consider these complex practical factors, which limits the applicability and effectiveness of the model to a certain extent. At the same time, the mechanical properties of engineering materials may also show differences due to a variety of factors such as the manufacturing process and the history of use, which may lead to deviations between theoretical models and practical applications. Therefore, when solving specific engineering problems, theoretical mechanics often needs to be cross-fertilised with other disciplines such as materials science, fluid mechanics, thermodynamics, etc., to form an integrated research approach. While this interdisciplinary integration provides a more comprehensive analytical perspective for the theoretical model, it also increases the complexity of the model and raises the difficulty of model building and analysis. Researchers need to have profound theoretical knowledge and rich practical experience in order to accurately capture the key factors in complex engineering problems and establish theoretical models that are both scientific and practical.

6. Trends in Theoretical Mechanics

The field of theoretical mechanics also needs to innovate and develop new theories and methods in the face of current challenges and limitations in theory and practice. In order to improve the accuracy and reliability of theoretical models, researchers need to combine theoretical mechanics with advanced computer techniques and experimental tools to address these challenges. For example, the non-linear behaviour of materials is a common phenomenon when dealing with practical problems. In order to reflect these behaviours more realistically, researchers can develop nonlinear finite element models, which are capable of integrating the elastic-plastic, fatigue and other nonlinear properties of the material to provide more accurate simulation results [115]. For elasto-plastic behaviour, researchers can also use the technique of incremental loading, which breaks down the entire loading process into a series of small incremental steps, allowing the researcher to calculate the response of the structure step by step [116]. This approach not only improves the stability of the calculations, but also captures in more detail the changes in the non-linear behaviour of the material during loading.

As a basic science with deep roots, theoretical mechanics is making great strides towards the development of cross-fertilisation with multiple disciplines. It not only occupies a central position in traditional engineering fields such as civil engineering, materials science, and mechanical engineering, but also plays an equally crucial role in emerging fields such as biomedicine and micro- and nanotechnology. Especially in the field of micro- and nanotechnology, the introduction of theoretical mechanics provides solid theoretical support and innovative methodology for research. By applying the principles of theoretical mechanics, researchers are able to gain a deeper understanding and prediction of mechanical behaviour at the nanoscale, which is essential for designing and optimising structures at the micro- and nanoscale [117]. These studies not only broaden the application scope of theoretical mechanics itself, but also inject new vigour and possibilities into the development of micro and nanotechnology. The integration of theoretical mechanics and biomedicine has given rise to the promising interdisciplinary discipline of biomechanics, which shows great potential in modelling the motion of living organisms, understanding the complex mechanical properties of biomaterials [118], and developing advanced medical devices. In the field of materials science, the application of theoretical mechanics promotes an in-depth understanding of the relationship between the microstructure and macroscopic properties of materials and provides theoretical guidance for the design and manufacture of high-performance materials.

Looking ahead, the trend of cross integration of disciplines will greatly expand the research horizons and application fields of theoretical mechanics, opening up new ideas and directions for solving complex engineering and scientific problems. As theoretical mechanics continues to advance and interdisciplinary integration advances, it will continue to play a cornerstone role in the development of engineering and science. The deepening development of theoretical mechanics, combined with interdisciplinary innovative thinking, will provide a solid theoretical foundation and effective solution strategies for solving complex problems in multiple fields. From macroscopic to microscopic, from traditional to cutting-edge, the principles and methods of theoretical mechanics will contribute to innovations and breakthroughs in science and technology at many levels. Through deep interdisciplinary cooperation, the research and application of theoretical mechanics will become more diversified and intelligent, contributing more wisdom and strength to the development of human society.

All in all, theoretical mechanics, as a bridge connecting different disciplines, still has a broad development prospect in the future. It will continue to play an irreplaceable role in promoting scientific progress, facilitating technological innovation and solving practical problems, opening up more possibilities for the development of human society.

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