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Review

A unified framework for equivalent single layer theories of composite beams, plates, and shells: a review

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Abstract: This article presents a comprehensive review of the unified framework for the equivalent single layer theories of composite beams, plates and shells. First, the approach to unified displacement fields has two main categories: one is to start from the assumption of linear displacement field variables. And the another is to start from the assumption of transverse strains and transverse displacement variable. In terms of unified shape functions, the shape function is defined as the function that exist in the transverse strain rather than the in-plane displacement variables. Moreover, the basic requirements for the choose of shape functions are also given and analysed in this paper. The unified shape functions are introduced in terms of the several functions and the unified polynomial model. In addition, a construction method for new shear strain shape functions is described in this review. Finally, the current research progress and future prospects are also given.

Keywords: Unified model; Equivalent single layer theory; Composite structures

1. Introduction

The composite beams, plates and shells are the basic units of ship and marine composite structures. For example, composite laminates and sandwich panels have many advantages,

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such as light weight, corrosion resistance, high strength, high damping, high designability, sound absorption, sound transmission and fatigue resistance [1–10]. Therefore, the static bending, vibration and buckling characteristics of composite beams, plates, and shells have been widely studied, and these characteristics often require the prediction of the static and dynamic characteristics of composite beams, plates, and shells by experimental, numerical, and theoretical methods.

The modeling methods of composite beams, plates and shells mainly include the three-dimensional elastic modeling method and the equivalent single layer theory. The three-dimensional elastic modeling is to discretize the displacement field of the composite plate and shell structures through the idea of finite element method and the three-dimensional representative volume elements. The static and dynamic results are calculated using suitable solution strategies. However, the three-dimensional refined modeling strategy brings high computational cost and response time, which is contrary to the concept of instantaneous prediction of structural digital twin, so the low-dimensional modeling method represented by the equivalent single layer theory has received wide attention. For composite thin plate structures, the effect of the transverse shear function on the prediction accuracy of the dynamic properties is negligible, however, for structures such as medium-thick and thick beams, plates, and shells, the transverse shear effect needs to be taken into account in order to avoid the loss of prediction accuracy. The selection of the transverse shear function is not strictly defined, as long as the free boundary conditions for shear stress are satisfied, so the selection of the transverse shear function is time-consuming and costly to conducted.

The best-known unified method in equivalent single layer theories is the Carrera unified formulation (CUF) which is first proposed by Carrera [11]. The improved CUF theories are suitable for analyzing the global stress distribution in composite laminated structures and their applications have been presented in a large number of publications [12–31]. In 2017, Abrate and Sciuva [32] compared the existing methods to classify the displacement field based on nonpolynomials, polynomials, and the number of unknown variables. The advantage of these methods is that it is easy to classify and extend the displacement field, but the fixed form is not conducive to the discovery of specially constructed displacement fields. In 2016, Nguyen et al. [33] fit all the shear strain shape functions with a unified polynomial, and then proposed a new higher-order shear deformation theory based on the unified formula. The advantage of this method is that it can unify all the shear shape functions in a simple polynomial form but the fitting accuracy may affect the prediction accuracy. In 2020, Nguyen et al. [34] classified and summarized the existing composite beam theories and proposed a unified composite beam theory. Li et al. [4] proposed a unified composite plate theory, which classified the existing theories into three categories from the expressions of shear strains, and new higher-order shear deformation theories can be proposed based on the unified plate theory.

This paper presents the first overview of the unified theory of composite beams, plates and shells, and the purpose of this paper is mainly to inspire the reader to construct new displacement fields through the current strategy of unified theory. This review paper is divided into four different parts. Section 1 describes some of the advantages of composite beams, plates and shells and their background in the field of marine and offshore engineering. Section 2 describes the unified displacement field and gives two different starting points for the unification of linear displacement variables and shear strains. In Section 3, the construction method of the unified shear function is presented. Section 4 summarizes recent research on the unified theory of composite beams, plates, and shells and provides guidance for future research.

2. The unified displacement fields

2.1 Starting with the displacement field variables

In terms of the kinematic assumptions, Abrate and Sciuva [32] used two broad categories (polynomials functions and non-polynomials functions) to express the displacement fields. In this paper, the displacements *u* and *v* are expanded as combined series of the transverse coordinate *z*. In terms of the equivalent single-layer plate theory, it is assumed that the generalized displacements (linear displacements, rotations, etc.) at each location point within the plate can be described by the reference displacements of the midplane, and reduces the computational dimensionality and the number of unknown displacement functions by the separation of variables method, which in turn improves the computational efficiency, and this method is also used in beams and shells. For example, in this section the displacements u, v, w in Cartesian system (x, y, z) are expanded into a mixed series of thickness coordinates z as follows:

$$
\begin{aligned}
u &= \sum_{i=0}^{N} u_i(x,y,t) z^i + \sum_{j=0}^{N} u_j(x,y,t) f_j(z) \\
v &= \sum_{i=0}^{N} v_i(x,y,t) z^i + \sum_{j=0}^{N} v_j(x,y,t) f_j(z) \\
w &= \sum_{i=0}^{N} w_i(x,y,t) z^i + \sum_{j=0}^{N} w_j(x,y,t) f_j(z)
\end{aligned}
\tag{1}
$$

where u_i, u_j, v_i, v_j, w_i, w_i are the unknown displacement functions of the physical middle surface, independent of the thickness coordinate z. The equivalent single layer theory is usually divided into two broad categories. In the first class, the displacement field is represented by a polynomial function in the thickness coordinate *z*, while in the second class, a non-polynomial function is used to describe it.

2.1.1 Polynomial displacement fields

Simple polynomials or orthogonal polynomials can be developed by expanding the displacement as a power series of the thickness coordinate *z* form [32]. The theory of simple polynomials is a special case of Eq. (1) when ignoring the second term in Eq. (2) is ignored.

$$u = u_i(x,y,t)z^i = u_0(x,y,t) + u_1(x,y,t)z + u_2(x,y,t)z^2 + u_3(x,y,t)z^3 + ...$$

$$v = v_i(x,y,t)z^i = v_0(x,y,t) + v_1(x,y,t)z + v_2(x,y,t)z^2 + v_3(x,y,t)z^3 + ... \tag{2}$$

$$w = w_i(x,y,t)z^i = w_0(x,y,t) + w_1(x,y,t)z + w_2(x,y,t)z^2 + w_3(x,y,t)z^3 + ...$$

The above equations can theoretically be extended infinitely (the number of terms *N* tends to infinity). However, this gives highly computational cost due to involving in an excessive number of unknown functions and in that case, the advantage of fast operations of equivalent single-level theories would be lost. Therefore, the highest power of the power series in the above equations is often restricted for simple modeling.

For beams, only the displacement unknowns *v* and displacement components *y* are removed on the basis of the above equation, while the other terms remain unchanged, so it can be considered that the equivalent single-layer beam theory is a special case of the equivalent single-layer plate theory, as follows.

$$u = u_i(x,t)z^i = u_0(x,t) + u_1(x,t)z + u_2(x,t)z^2 + u_3(x,t)z^3 + ...$$

$$w = w_i(x,t)z^i = w_0(x,t) + w_1(x,t)z + w_2(x,t)z^2 + w_3(x,t)z^3 + ... \tag{3}$$

The polynomial theories are usually classified into Classical plate theory, First-order shear deformation theory, Higher order shear deformation theory), etc. Before introducing these different polynomial theories, we first The concept of {*m*,*n*} theory is explained. The parameter m in the {*m*,*n*} polynomial theory represents the highest order of *z* in the expression of the in-plane displacement variable *u* and the highest power of *z* in the expression of the variable *v*. The parameter n represents the highest power of *z* in the expression of the thickness variable *w*. in the polynomial theory. For example, the polynomial theory $u = u_0 + u_1 z + u_2 z^2$, $v = v_0 + v_1 z + v_2 z^2$, $w = w_0$ would be the {2,0} theory.

2.1.1.1 Classical plate theory

In continuum mechanics, a large number of plate and shells theories have been proposed and developed in decades. The classical plate theory is first proposed by Kirchhoff [35] in 1850 based on his assumptions and then developed by Love [36] in 1888. They assume that a plane section normal to the mid-surface of plates remain plane and keeps normal during and after deformation and thickness of the plates does not change in this process. It is worthwhile to mention that the classical plate theory is only applicable to thin plates for the neglect of transverse shear deformation effects as well as the moment of inertia. When it comes to the moderate thick plates and thick plates, the predicted deflections are underestimated and natural frequencies and buckling loads are overestimated [37–39].

According to the displacement field listed in Eq. (2), the following strains can be obtained as

$$\begin{aligned}
\varepsilon_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = (u_1 + \frac{\partial w_0}{\partial x}) + (2u_2 + \frac{\partial w_1}{\partial x})z + (3u_3 + \frac{\partial w_2}{\partial x})z^2 + \ldots \\
\varepsilon_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = (v_1 + \frac{\partial w_0}{\partial y}) + (2v_2 + \frac{\partial w_1}{\partial y})z + (3v_3 + \frac{\partial w_2}{\partial y})z^2 + \ldots \\
\varepsilon_{zz} &= \frac{\partial w}{\partial z} = w_1 + 2w_2 z + 3w_3 z^2 + \ldots
\end{aligned} \tag{4}$$

Based on the assumptions that the thickness of the plates does not change during and after the deformation, the strain ε_{zz} is equal to zero throughout the thickness of the plate. In that case, $w_1=w_2=w_3=0$.

In addition, it is assumed that a plane section perpendicular to the mid-surface remains plane, normal during and after deformation. Eq. (4) indicates that transverse shear strains ε_{xz} and ε_{yz} are neglected and independent of z ($\varepsilon_{xz}=\varepsilon_{yz}=0$). Therefore, based on classical plate theory (or Kirchhoff-Love assumptions), the deformation of the plate is described as follows

$$u_1 = -\frac{\partial w_0}{\partial x} \; u_2 = u_3 = \ldots = 0$$

$$u = u_0(x,y,t) - \frac{\partial w_0}{\partial x} z, \;\; v = v_0(x,y,t) - \frac{\partial w_0}{\partial y} z, \;\; w = w_0(x,y,t) \tag{5}$$

2.1.1.2 The first-order shear deformation theory

The first-order shear deformation theory (also named Reissner–Mindlin plate theory) proposed by Reissner [40] and Mindlin [41] is the first and simplest theory to takes into account transverse shear deformations throughout the plate thickness, it is assumed that the plane section perpendicular to the mid-surface remains plane but not necessarily perpendicular to the mid-surface, so it could be considered that this theory is also an extension of classical plate theory mentioned in the last section. Though FSDT surmounts some limitations of CPT, it also has its own drawback: the linear displacement variation throughout the thickness leads to constant transverse shear strains and stresses across the plate thickness. However, this phenomenon violates: (1) the stress-free boundary conditions on the top and bottom surface of plates; (2) parabolic distribution of transverse shear stresses throughout the thickness. So, the integral of transverse shear stresses based on FSDT assumptions are greater than the real values, therefore, a shear correction factor (less than 1) is required to properly predict the real mechanical behavior of plates and give acceptable analytical results for moderately thick and thin plates.

In terms of the shear correction factor which should be carefully chosen in all kinds of cases, though it is hard to pick up. A constant value of *K*=5/6 is commonly used in numerical studies. Timoshenko [42] presented a shear correction coefficient which is dependent with the Poisson ratio *v*.

$$K = \frac{5+5v}{6+5v} \tag{6}$$

Efraim and Eisenberger [43] then presented a shear correction factor for FGM plates as

$$K = \frac{5}{6-(v_c V_c + v_m V_m)} \tag{7}$$

where v_c and v_m represent the Poisson ratios of pure ceramic and pure metal, V_c and V_m denote the volume fractions of ceramic and metal in the entire thickness of the plate.

Effects of the shear correction coefficients on the natural frequency of the FG plates is studied by Zhao et al [44]. They calculated the natural frequencies with different values of shear correction coefficients with a constant value of 5/6, and compared those results with a HSDT results reported by Matsunaga [45]. The results show that the modified shear correction coefficients yield more accurate result than the constant value of shear correction coefficients does for a moderately thick (*a*/*h*=10) and thick (*a*/*h*=5) square FG plates. For a thin FG plate, there is no discernible differences found.

The modified first-order shear deformation theory (MFSDT) proposed by Thai et al. [46–48] is an extension of the classical first-order shear deformation plate theory. In MFSDT, the transverse displacement *w* is divided into bending and shear parts ($w=w_b+w_s$) and it is assumed that $\varphi_x = -w_{b,x}, \varphi_y = -w_{b,y}$. The following displacement field is obtained:

$$u(x,y,z)=u_0(x,y)-zw_{b,x},\ v(x,y,z)=v_0(x,y)-zw_{b,y},\ w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{8}$$

2.1.1.3 The second shear deformation theory

The displacement field of the second-order shear deformation theory has the highest order of two. For example, the in-plane displacement variables including linear strains and shear strain in the following equation are all quadratic.

$$u = u_0 + zu_1 + z^2 u_2,\ v = v_0 + zv_1 + z^2 v_2,\ w = w_0$$

$$\varepsilon_x = u_{0,x} + zu_{1,x} + z^2 u_{2,x},\ \varepsilon_y = v_{0,x} + zv_{1,x} + z^2 v_{2,x},$$

$$\gamma_{xy} = u_{0,y} + v_{0,x} + z(u_{1,y} + v_{1,x}) + z^2(u_{2,y} + v_{2,x}), \tag{9}$$

$$\gamma_{xz}(x,y,z) = u_1 + 2zu_2 + w_{0,x}, \gamma_{yz}(x,y,z) = v_1 + 2zv_2 + w_{0,y}$$

In addition, the readers can tell from the Eq. ($\gamma_{xz} = u_1 + 2zu_2 + w_{0,x}, \gamma_{yz} = v_1 + 2zv_2 + w_{0,y}$) that the transverse shear stresses are linear rather than parabolic throughout the thickness. To be specific, $\gamma_{xz}(x,y,\pm h/2) = u_1 \pm hu_2 + w_{0,x}, \gamma_{yz}(x,y,\pm h/2) = v_1 \pm hv_2 + w_{0,y}$ are not always equal to zero unless we assume that

$$\begin{aligned} &\gamma_{xz}(x,y,h/2) = u_1 + hu_2 + w_{0,x} = 0 \\ &\gamma_{xz}(x,y,-h/2) = u_1 - hu_2 + w_{0,x} = 0 \\ &\gamma_{yz}(x,y,h/2) = v_1 + hv_2 + w_{0,y} = 0 \\ &\gamma_{yz}(x,y,-h/2) = v_1 - hv_2 + w_{0,y} = 0 \end{aligned} \tag{10}$$

In that case, $u_1 = -w_{0,x}$, $v_1 = -w_{0,y}$ and $u_2 = v_2 = 0$. Therefore, the SSDT that satisfies the stress-free boundary condition degrades to the CPT in Eq. (5). Therefore, it could be considered that the SSDT is also an extension of Kirchhoff–Love plate theory that does not take into account stress-free condition on the top and bottom surfaces of the plate.

2.1.1.4 The third-order shear deformation theory

Similar to FSDT and SSDT, the "third" in the third-order shear deformation theory refers to the highest number of in-plane transverse displacements *u*, *v* with respect to *z*. It can be divided into [3,*n*] theories based on the construction of transverse displacement w, where n is the highest number of times z in transverse displacement w. For instance, in the [3,0] theory, the thickness stretching effect is not considered and $\varepsilon_{zz}=0$. In the [3,1] theory, $\varepsilon_{zz} = w_1$ is a constant, which represents that the thickness stretching is constant throughout the thickness direction. In the [3,2] theory, $\varepsilon_{zz} = w_1 + 2z w_2$ is a constant representing that the strain is linear throughout the thickness direction. In the [3,3] theory $\varepsilon_{zz} = w_1 + 2zw_2 + 3z^2w_3$, which has a quadratic variation throughout the thickness of the beams, plates and shells. For example, Ref. [49] describes how to construct the [3,2] theory and then eliminate the redundant unknown functions by means of the stress-free condition, and finally the simplified displacement fields are obtained.

2.1.1.5 Other polynomial shear deformation theories

Other higher order shear deformation theories improve accuracy by increasing the number of displacement field variables, but also bring additional computational time.

According to the Ref. [50], the computational accuracy converges as the order increases. The fourth-order shear deformation theory is more accurate than the third-order shear deformation theory in terms of out-of-plane shear strain, but the accuracy of transverse displacement is almost the same. Therefore, higher orders are not necessary because the results obtained by the 4th order already almost overlap with the exact solution.

2.1.2 Non-polynomial displacement fields

Non-polynomial terms (such as shear functions $f(z)$ in the following equation) are introduced in the displacement fields for more accurate predicted results.

$$u = u_0 - z\frac{\partial w_0}{\partial x} + f(z)\theta_x,\ v = v_0 - z\frac{\partial w_0}{\partial y} + f(z)\theta_y,\ w = w_0 \tag{11}$$

Nonpolynomial displacement fields are often categorized by the number of unknowns and the shape of $f(z)$ [32]. The number of unknowns is not discussed in detail in this paper, and the shape of $f(z)$ is discussed will be discussed in the next section.

2.2 Starting with the transverse displacements and out-plane shear strains

Nguyen et al. [34] proposed a novel unified beam model for laminated beams and they developed a unified displacement field which can be recovered to that of existing shear deformation beam theories reported in the published literature. Only 2D shear deformation theory without thickness stretching effects is studied in Ref. [1] for simplicity.

The stress-strain relationships of the beams are given by:

$$\gamma_{xz}(x,z) = \frac{1}{G}\sigma_{xz}(x,z) \tag{12}$$

where G is shear modulus. The linear elastic relationships of strains and displacements are expressed by

$$\begin{gathered} \gamma_{xz}(x,y,z) = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ u = u_0 - z\frac{\partial w_0}{\partial x} + f(z)\frac{Q_x(x)}{G},\ w = w_0 \end{gathered} \tag{13}$$

When $Q_x(x) = G\theta_0(x),\ u = u_0 - zw_{0,x} + f(z)\theta_0,\ w = w_0$

When $Q_x(x) = \frac{5Gh}{6}(\theta_0(x) + w_{0,x}),\ u = u_0 + [\frac{5hf(z)}{6} - z]w_{0,x} + \frac{5hf(z)}{6}\theta_0(x)$ in Refs. [51–53].

Subsequently, Li et al. categorized the out-plane shear strains of composite plates into three types based on the existing literature. The first type contains one displacement variable Q_{1x}. The second type contains two displacement variables Q_{1x} and Q_{2x}, and the same shear function $g(z)$. The third type contains two displacement variables and different shear strain shape functions. It is easy to notice that the third type can be used as a unified form.

Type 1: $\gamma_{xz}(x,y,z)=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=g(z)Q_{1x}(x,y)$

Type 2: $\gamma_{xz}(x,y,z)=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=g(z)[Q_{1x}(x,y)+Q_{2x}(x,y)]$

Type 3: $\gamma_{xz}(x,y,z)=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=g_1(z)Q_{1x}(x,y)+g_2(z)Q_{2x}(x,y)$

Integrating the equation in Type 3 along the thickness coordinate z yields an expression for the line displacement u. Similarly, one can integrate over γ_{yz} to obtain an expression for the line displacement v. Thus, the unified displacement fields are expressed as:

$$u=u_0-z\frac{\partial w}{\partial x}+f_1(z)R_{1x}(x,y)+f_2(z)R_{2x}(x,y)$$

$$v=v_0-z\frac{\partial w}{\partial y}+f_1(z)R_{1y}(x,y)+f_2(z)R_{2y}(x,y) \tag{14}$$

$$w=w$$

where the transverse displacement w is user-defined and is independent of thickness coordinate z if the thickness stretching effect is ignored, and if the thickness stretching effect is considered for thick plate and shells, the transverse displacement should be expressed as a function of thickness coordinate z.

For instance, when $w=w_b, Q_{1x}(x,y)=\theta_x(x,y)$, the popular displacement field can be obtained [7,39,54–65] as follows.

$$u=u_0-zw_{0,x}+f(z)\theta_x(x,y),\ v=v_0-zw_{0,y}-f(z)\theta_y(x,y),\ w=w_0 \tag{15}$$

3. Shear strain shape functions

Many researchers used different type of shear strain shape functions (trigonometric, hyperbolic, exponential and combination) to describe shear deformation effect.

Table 1. Shear strain shape functions in different theories.

Theories	**References**
Trigonometric	[60] [66-72]
Hyperbolic	[55] [62] [73-76]
Exponential	[77-97]
Combination	[39] [59] [98]

In order to construct new shear strain shape functions, Li [99] proposes a construction method. First, any even function $A(z/h)$ with an independent variable z/h is given, before shifting the even function a distance of $A(0.5)$. Then, resizing the function Along the horizontal coordinate axis with a ratio of 1/[$A(0)$-$A(0.5)$] to satisfy the basic conditions ($f'(0)=1$). Finally, the new shear strain shape function $f(z)$ can be obtained by integrating the scaled $f'(z)$ along the thickness direction, as shown in Fig. 1.

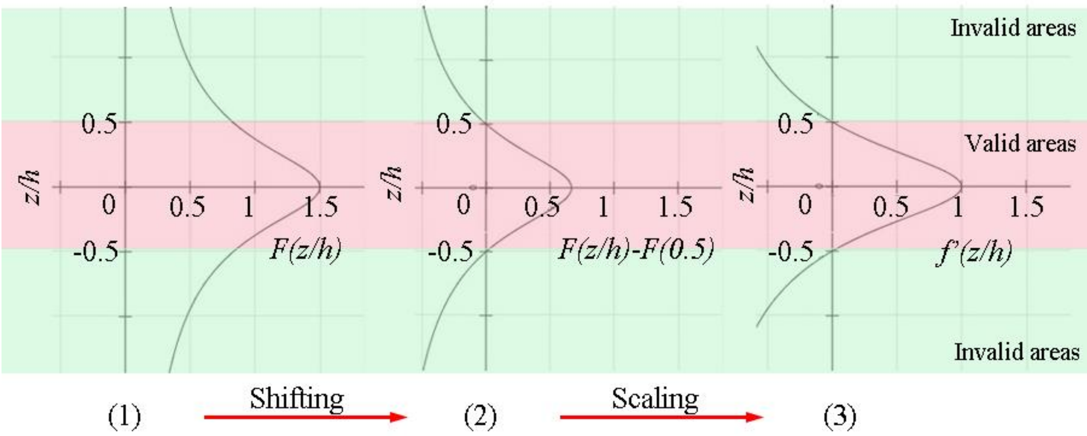

Fig. 1. The construction method for new shear shape functions [99].

To unify different types of shear strain shape functions, Nguyen et al. [33] proposed a unified polynomial formulation to unify all existing shape functions by fitting some critical points in the previous functions. However, some potential shape functions might not be easy to find due to the limit of these types of functions. Therefore, in future research, the location of these control points can be corrected using methods such as deep learning to obtain more accurate shear strain shape functions.

4. Conclusion

This review provides an overview of the unified framework of equivalent single layer theory for composite beams, plates and shells in terms of displacement fields and shear strain shape functions, respectively. This helps the readers to get more accurate displacement fields using the mentioned unified theory. For composite beams, plates and shells, traditional methods such as classical plate theory and first order shear deformation theory are sufficiently accurate. For thick ones, transverse shear effects and thickness stretching effects need to be taken into account, so more accurate equivalent single layer theories are studied. Current references focus on obtaining predictive solutions for bending, free vibration, and buckling loads and comparing them with elastic solutions to assess the accuracy of the applied theory. The literature show that the accuracies of the majority of theories are satisfactory, so further investigation is needed only for new materials and new structural forms. In addition, the fourth-order polynomial displacement field has a good convergence and more orders are not suggested. The non-polynomial displacement field relies on the search for the shear function. However, the mathematical functions are still limited for better shear strain shape functions. Therefore, the more accurate description of the shear function may depend on the use of new technologies and methods such as machine learning.

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