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Article

Study on the Predictive Accuracy of Different Three-Dimensional Rock Strength Criteria

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Abstract: This study investigates the predictive accuracy of different three-dimensional rock strength criteria. The research focuses on the comparison between ZZ criteria and HBWW criteria against the HB, PH, and GP criteria. The results demonstrate that the ZZ and HBWW criteria exhibit significant advantages over the HB, PH, and GP criteria. The primary reason for this is that the HB criterion does not consider the influence of the intermediate principal stress, while the PH and GP criteria, although considering the intermediate principal stress, do not distinguish the effect of the Lode angle on rock strength, leading to larger prediction errors.

Keywords: Rock strength; Three-Dimension; Criteria; Predictive accuracy; Prediction errors

1. Introduction

Rock mechanics is the discipline that studies the deformation, movement, and failure of crustal rocks under various conditions. According to the 1966 definition by the Rock Mechanics Committee of the National Academy of Sciences in the United States, rock mechanics is a theoretical and applied science that examines the mechanical behavior of rocks, exploring their responses to the force fields within their surrounding physical environments. It is particularly noteworthy that, due to the extreme heterogeneity of rock masses, relying solely on point strength criteria to determine the allowable stress and safety factors for design leads to significant uncertainties in predicting the stability and reliability of entire rock engineering projects. In practice, rock mass strength criteria are merely theoretical estimates of reliability in rock engineering design. Given the extreme complexity of rock masses, it is difficult to definitively classify any criterion as either good or bad, as each has its applicable conditions. Therefore, it is impractical for the field of rock mechanics to spend too much effort on this issue.

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Rock strength criteria are one of the most fundamental problems in rock mechanics research. These criteria have widespread applications, not only in energy development, geophysical and geological exploration, but also in the theoretical analysis, practical application, and efficient utilization of materials in these fields. While strength criteria may seem simple, they are, in fact, very complex, integrating aspects from physics, mathematics, and natural sciences. To date, rock mechanics experts have proposed over a hundred criteria and models, and have conducted extensive theoretical research and experimental validation. However, none of these criteria and models can be universally applied to all rock mechanics problems and related engineering issues. In the field of engineering mechanics, rock mechanics strength criteria serve as one of the important standards for design and construction, structural safety, and assessing rock failure. Over the past decades, experts have proposed various strength criteria suitable for rock engineering, such as the Mohr-Coulomb criterion, Hoek-Brown criterion, maximum tensile stress criterion, and Griffith criterion. Although some criteria are widely used in rock strength systems, they all overlook the effect of the intermediate principal stress on rock strength. Thus, rock mechanics experts have derived true triaxial theoretical strength criteria to study this influence. Selecting a reasonable yield criterion is an important issue, as is finding a unified yield criterion applicable to various materials and establishing relationships between different yield criteria.

Extensive research has been conducted domestically and internationally on the characteristics and predictive accuracy of rock strength criteria. Early studies on material strength theories began with metallic materials. For instance, Von Mises proposed the Mises yield criterion, which states that failure occurs when the octahedral shear stress within a material reaches its yield strength. This theory applies to materials with identical tensile and compressive strengths and does not consider the impact of hydrostatic pressure on failure. Rock failure is primarily controlled by deviatoric stress. Drucker and Prager modified this theory, proposing the Drucker-Prager criterion, which, however, cannot describe the differences in strength along different meridians of the rock. The DP criterion overestimates the effect of the intermediate principal stress on ultimate strength, making it applicable only to materials with identical tensile and compressive strengths, and its use in rock materials has gradually decreased. Unified strength theory has been a long-sought goal in research, often regarded as an unsolvable problem. The MC criterion proposed in 1900 forms the inner boundary of the Drucker hypothesis, while the double-shear strength theory proposed by Yu Maohong in 1985 serves as its outer boundary. Yu Maohong (1991) established the unified strength theory based on the Drucker hypothesis and its corresponding convexity of loading surfaces. The single-shear and double-shear strength theories objectively define the two boundaries of convex strength theories, with a wide area between them covered by a series of ultimate surfaces defined by the unified strength theory proposed in 1991. The unified strength theory can be either non-convex or convex depending on the value of parameter b. Various failure criteria are special cases or linear approximations of this theory, describing the failure characteristics of materials with different intermediate principal stress effects. Shen Zhujiang (1995) categorized shear strength theories into three major series: generalized single shear theory, generalized double shear theory, and generalized triple shear theory. In stress space, the ultimate surface of the generalized nonlinear unified strength theory is bounded by the generalized HB strength criterion on the inner side and the generalized double anti-shear strength criterion on the outer side. A series of new ultimate surfaces cover the entire area between these two boundaries. As the unified strength theory can only linearly approximate nonlinear criteria and cannot completely unify them, Yu Maohong (2007) developed a dodecahedral stress model, expanding the unified strength theory into a more inclusive form that encompasses both linear and nonlinear criteria. Most strength theories have similar shapes for their ultimate traces on the π plane, and these traces lie within the boundaries of single-shear and double-shear strength theories. On the π plane, the material's ultimate trace exhibits three-axis symmetry, and the shape of the yield surface within the 0~60° range can define the entire 360° range on the π plane. Coulomb proposed that failure occurs along a specific plane due to sliding, and suggested considering both the cohesion and the normal force-induced friction on the sliding plane. The main drawback of the Mohr-Coulomb criterion is its lack of consideration for the intermediate principal stress, which is an interesting topic for further development. As the value of the intermediate principal stress increases, the internal friction angle of dense sand can increase by up to 5°. Yu Maohong (1988) divided numerous strength theories into three major series: Single Shear Strength Theory (SSS), Double Shear Strength Theory (TSS), and Octahedral Shear Stress Strength Theory (OSS). On the π plane, SSS theory forms the lower limit (inner boundary) of all possible convex-shaped ultimate surfaces, while OSS theory, a nonlinear function, lies between SSS and TSS theories. TSS theory is a new series of strength theories, which is also a linear function and forms the upper limit (outer boundary) of convex-shaped ultimate surfaces on the π plane. Yu Maohong (2014) summarized the three major series of strength theories, the unified yield criterion, the unified strength theory, and other various strength theories, discussing the relationships among them to provide a method for reasonably selecting failure criteria in research and engineering applications. The desired strength theory should reflect the basic characteristics of rocks, such as different tensile and compressive strength properties, the effect of hydrostatic pressure, the effect of normal stress, and the effect of the intermediate principal stress. It should also match existing experimental data well, have physical significance, include all independent stress components, and be expressed with simple equations. It should be suitable for various stress states and different types of materials, reflecting the material's response with the maximum number of material parameters, incorporating well-known failure criteria as special cases and linear approximations, and establishing relationships among various failure criteria. Jiang and Pietruszczak (1988) examined the convexity of a yield locus on the π plane using appropriate analytical criteria for several shape functions proposed by WW, GA, and BC, and proposed two new, simple, and convex forms of the shape functions $g(\theta)$. Yu Maohong and Liu Fengyu (1990) smoothed the corners of the generalized double-shear stress criterion, obtaining two new smooth, convex models of the generalized double-shear stress criterion corners. The shape function $g(\theta)$ defines the ultimate trace of the corner model on the π plane. The smoothed shape function should be smooth, convex, symmetric with respect to the principal stress axis, and capable of reflecting the effect of the intermediate principal stress, matching both the tensile and compressive meridians of the ultimate surface. Yu Maohong considered the effect of the intermediate principal stress and modified the Tresca and MC criteria, respectively establishing the double-shear stress criterion and the generalized double-shear stress criterion, proving that the generalized double-shear stress criterion and the MC criterion are the upper and lower limits of all possible yield surfaces. Gudehus and Argyris (1987) proposed a new function on the π plane, which is smooth and approaches the MC yield function without any sharp angles. Shi Shuzhao and Yang Guanghua's research found that this function becomes concave on the π plane when the internal friction angle of rock exceeds 22.02°, and they proposed a yield function that maintains a convex, smooth surface without sharp angles. Lin and Bažant (1986) evaluated three shape functions in the deviatoric stress cross section, indicating that Willam and Warnke's function should be preferred despite its more complicated form. Rock strength criteria increase nonlinearly with increasing hydrostatic pressure on the meridian plane, but the increase rate first grows and then diminishes, ultimately tending to zero. Li Xiulei et al. (2021) constructed strength criteria that conform to the nonlinear characteristics of rock strength on the meridian plane and considered the effect of intermediate principal stress by introducing intermediate principal stress and Lode stress parameters, establishing a new nonlinear true triaxial strength criterion for rocks. This criterion satisfies the nonlinear characteristics of the meridian plane, but its ultimate trace on the π plane does not meet the convexity requirement of the yield surface and is not smooth in the tension-compression transition zone.

Regarding the predictive performance of existing rock strength criteria for rock strength experimental data, Li Bin et al. (2016) suggested that current rock strength criteria overestimate the strength of rocks under high confining pressures. They introduced the concept of critical state confining pressure and improved the MC and HB strength criteria based on this concept. Their derivation showed that when the confining pressure gradually increases to the rock's critical state, the deviatoric stress remains constant, and the critical fracture angle of the rock is 45°. You Mingqing (2009) proposed an exponential strength criterion, a direct function of the principal stresses, which fits well and is applicable over a wide range of stress. The exponential strength criterion harmonizes with the strength under unloading confining pressure, low confining pressure, and high confining pressure, as compared to the MC criterion, HB criterion, and quadratic polynomial criteria. You Mingqing (2010) found that using the least squares method to determine the parameters in the strength criteria, larger errors in data can lead to the overall fitting curve deviating from most normal data. He suggested using the minimum absolute value of the fitting deviation to obtain the undetermined parameters in the strength criteria. Previous studies believed that the significant deviation between the single-parameter parabolic criterion and actual experimental data might be caused by using the least squares method to fit the undetermined parameters. The fitting accuracy of single-parameter criteria can surpass that of double-parameter criteria. Mogi (1967) conducted extensive research on the effect of intermediate principal stress on rock failure. He found that the increase in strength at failure is proportional to and smaller than the confining pressure, and the angle between the failure plane and maximum principal stress significantly reduces with increasing second principal stress.

Carter et al. (1991) modified the Hoek-Brown criterion by adding a third parameter to account for low tensile strength, resulting in the modified HB criterion closely following the strength data in the tension-low confining pressure region. You (2010) pointed out that the minor principal stress is the key factor influencing the mechanical properties of rock, and that the conventional triaxial strength criterion is the basis of any true triaxial criterion. Furthermore, the four parameters embedded in the exponential strength criterion can be determined by the strength data under conventional triaxial compression and extension for true triaxial strength prediction. You (2010) indicated that the exponential strength criterion is suitable for describing the strength data for rocks at both brittle fracture and ductile failure, with the differential stresses being approximately constant at high confining pressures. Singh et al. (2011) modified the Mohr-Coulomb criterion by employing Barton's critical state concept for rocks. In further research, the critical confining pressure equals the unconfined compressive strength (UCS) of the intact rock. The authors extended the criterion to jointed rocks, which are anisotropic in nature, to assess the effect of minor and intermediate principal stress on the strength of the jointed rock mass. Further studies suggested that the critical state concept is also applicable to jointed rocks. Yang et al. (2018) compared the nonlinear unified strength criterion, which utilizes the unified strength theory and HB criterion, with other 3D strength criteria forms of HB, concluding that the nonlinear unified strength criterion is superior. Zan Yuewen et al. (2002) proposed a nonlinear unified strength criterion that considers the significant difference between rock tensile and compressive strength, as well as the interval effect of intermediate principal stress. The undetermined parameters in this criterion can be derived from the HB criterion and conventional triaxial experimental data. Research results indicate that the intermediate principal stress affects the strength of different rock types differently: soft rocks show a smaller effect of intermediate principal stress, while hard rocks show a larger effect. Zan Yuewen et al. (2013) established a generalized nonlinear unified strength theory based on the double shear model and the generalized HB strength criterion. The ultimate surface of this theory covers the entire convex region, and the generalized HB criterion and the generalized nonlinear double shear strength theory are special cases of this theory. Zan Yuewen et al. (2002) proposed a nonlinear unified strength criterion for brittle failure of rocks within a certain stress range (confining pressure < uniaxial strength). To address the ductile failure characteristics and nonlinear stress relationships of rocks under high hydrostatic pressure, Zan Yuewen et al. (2004) proposed a nonlinear unified strength theory under high hydrostatic pressure conditions. You Mingqing (2013) pointed out that the unified strength theory cannot describe the nonlinear increase in the minimum principal stress for triaxial compression and tensile strength, nor its trend with intermediate principal stress changes. Its nonlinear form uses two implicit functions in segments, making actual calculations difficult and particularly deviating for low confining pressure experimental data, overestimating rock strength, and potentially causing disasters in engineering applications. The linear unified strength theory with three parameters shows large fitting deviations for true triaxial strength, failing to describe the strength trend with intermediate principal stress changes. Sriapai et al. (2013) tested the true triaxial compressive strength of Maha Sarakhan salt, concluding that the elastic parameters of the salt tend to be independent of σ2 for the applied stress range. The effect of σ2 on salt strength can be best described by the modified Wiebols-Cook criterion compared to the power law of Mogi, modified Lade, and 3D HB criteria. Sharpe (2017) tested rock failure with a dog-bone specimen, where the minor principal stress is tensile and the intermediate and major principal stresses are compressive, using these results to evaluate four criteria: MC, Paul-MC, HB, and Fairhurst (Fh). Studies indicated that Fh provided the best overall fit because it is nonlinear and contains a tension cut-off. Li Xiulei et al. (2021) classified proposed three-dimensional rock strength criteria into two major categories: those constructed based on strength theory and experimental data, and those constructed by modifying conventional triaxial strength criteria to account for intermediate principal stress in the tensile-compressive transition zone. Current research on three-dimensional rock strength criteria mainly focuses on the HB criterion.

The selection of rock strength criteria is a key factor in determining the stability of geotechnical engineering. Some scholars suggest that the choice of strength criteria is more important than the study of the criteria themselves. This study, based on the least squares method, used five types of Hoek-Brown criteria to fit 32 sets of true triaxial rock strength experimental data, comparing and analyzing the fitting accuracy of different rock strength criteria. The research results contribute to the optimal selection of strength criteria.

2. Rock strength models and datasets

2.1 Rock strength criteria

2.1.1. Hoek-Brown criterion

In 1980, Hoek and Brown proposed the well-known Hoek-Brown empirical criterion for intact rock, as shown in Equation 1,

$$\sigma_1 - \sigma_3 = \sqrt{m_i \sigma_c \sigma_3 + \sigma_c^2} \quad (1)$$

In this equation, m_i is a parameter describing the lithology of intact rock cores, and σ_c is the uniaxial compressive strength of the rock, which is generally obtained through laboratory testing.

2.1.2. Pan-Hudson criterion

Pan and Hudson proposed a three-dimensional rock strength criterion based on the Hoek-Brown criterion's Mohr-Coulomb failure envelope and its corresponding Mohr circle in 1988. This criterion, also known as the Pan-Hudson (PH) criterion, averages the inner and outer Mohr circles of the hexagonal yielding surface on the π-plane of the Hoek-Brown criterion. It uses the variables effective mean stress I_1 and the second stress invariant J_2. The expression for this criterion is given by Equation 2,

$$\frac{3}{\sigma_c} J_2 + \frac{\sqrt{3}}{2} m_i \sqrt{J_2} - \frac{m_i I_1}{3} = \sigma_c \quad (2)$$

2.1.3. Generalized Priest criterion

Based on the DP (Drucker-Prager) criterion and the HB (Hoek-Brown) criterion, which should yield the same uniaxial compressive strength under uniaxial compression stress conditions, Priest proposed a three-dimensional strength criterion in 2015 known as the generalized Priest criterion. This criterion combines the Drucker-Prager and Hoek-Brown criteria and is expressed as Equation 3,

$$\frac{3}{\sigma_c}J_2+\frac{\sqrt{3}}{3}m_i\sqrt{J_2}-\frac{m_iI_1}{3}=s\sigma_c \quad (3)$$

2.1.4. Zhang-Zhu criterion

A large amount of experimental data has demonstrated that the Hoek-Brown criterion provides good fitting accuracy for rock strength under triaxial compression stress conditions. Therefore, a three-dimensional rock strength criterion based on the HB (Hoek-Brown) criterion should have a yield surface on the π-plane that connects seamlessly with the HB criterion under triaxial compression stress conditions. Based on the Mogi-Coulomb criterion, which assumes that the normal stress on the failure plane is independent of the second principal stress, Zhang and Zhu proposed a new three-dimensional HB criterion in 2007. They used the average principal stress $\sigma_{m,2}$ instead of the variable $I_1/3$ from the Pan-Hudson criterion. This criterion is also known as the Zhang-Zhu criterion,

$$\frac{3}{\sigma_c^2}J_2+\frac{m_i}{2\sqrt{3}}\left(3+2\sin\theta\right)\frac{\sqrt{J_2}}{\sigma_c}-\left(\frac{m_iI_1}{3\sigma_c}+1\right)=0 \quad (4)$$

2.1.5. Three-Dimensional Approximate Criterion for the Hoek-Brown Criterion

Using the variables on the π-plane, when the hydrostatic pressure is fixed, the distance from the hydrostatic pressure axis to any point on the yield curve of the HB (Hoek-Brown) criterion on the π-plane is given by Equation 5. When the Lode angle is 0° and 60°, we can respectively obtain the distances from the hydrostatic pressure axis to the yield curve under triaxial compression and triaxial tension stress states, as shown in Equations 5 and 6,

$$r_c=\frac{1}{3\sqrt{6}}\left(-m\sigma_c+\sqrt{m^2\sigma_c^2+12\sqrt{3}m\sigma_c\xi+36\sigma_c^2}\right) \quad (5)$$

$$r_t=\frac{2}{3\sqrt{6}}\left(-m\sigma_c+\sqrt{m^2\sigma_c^2+3\sqrt{3}m\sigma_c\xi+9\sigma_c^2}\right) \quad (6)$$

The ratio of the triaxial tensile strength to the triaxial compressive strength based on the Hoek-Brown criterion can be obtained as shown in Equation 7,

$$k\left(\xi\right)=\frac{-2m\sigma_c+2\sqrt{m^2\sigma_c^2+3\sqrt{3}m\sigma_c\xi+9\sigma_c^2}}{-m\sigma_c+\sqrt{m^2\sigma_c^2+12\sqrt{3}m\sigma_c\xi+36\sigma_c^2}} \quad (7)$$

To eliminate the six singularities on the π-plane of the Hoek-Brown criterion and ensure convexity, Lee et al. (2012) replaced the Lode angle shape function of the HB criterion with the Lode angle shape function developed by Willam and Warnke (Equation (8)). This approach resulted in a smooth and convex three-dimensional Hoek-Brown strength criterion,

$$g_{ww}(\theta)=\frac{2\left(1-k^2\right)\cos\left(\theta-(\pi/3)\right)+(2k-1)\sqrt{4\left(1-k^2\right)\cos^2\left(\theta-(\pi/3)\right)+5k^2-4k}}{4\left(1-k^2\right)\cos^2\left(\theta-(\pi/3)\right)+(2k-1)^2} \quad (8)$$

2.2 Strength datasets

Currently, true triaxial strength testing equipment for rocks remains relatively uncommon, posing challenges for experimental studies. Despite numerous true triaxial tests conducted worldwide over the past few decades, a significant limitation in studying rock behavior under three-dimensional stress remains the insufficient availability of comprehensive true triaxial test data to validate theoretical and empirical rock failure models. In previous validation studies of three-dimensional rock strength criteria, the selected criteria were often not comprehensive enough or based on limited true triaxial strength experimental data, potentially leading to incomplete research conclusions and even biases. This study compiles and references 32 sets of true triaxial strength experimental data from existing literature, as shown in Table 1.

Table.1 Summary of True Triaxial Rock Strength Experimental Data

Rock types	Experimental number/group	Minimum confining pressure /MPa	Maximum confining pressure /MPa	Data sources
Aghajari sandstone	47	5	40	Bahrami et al. (2017)
Asmari Limestone	33	20	80	
Chaldoran Metapelite	19	20	80	
Hormoz Salt	36	5	30	
Inada Granite	44	0	200	
Jahrom Dolomite	53	20	140	
Jolfa Marble	36	5	50	
Karaj Andesite	20	20	60	

Karaj Trachyte	34	40	100	
Mahalat Granodiorite	29	20	100	
Naqade Amphibolite	33	20	100	
Orikabe Monzonite	38	0	200	
Pabdeh Shale	32	20	80	
Shahr-e babak Hornfels	20	20	60	
shourijeh Siltstone	31	10	80	
Sandstone-Rukhaiyar	26	0	10	Rukhaiyar and Samadhiya (2017)
Westerly Granite	45	0	100	Haimson and Chang (2000)
Mizuho Trahchyte	31	0	100	Mogi, K. (2006)
KTB amphibolite	42	0	150	Lee et al. (2012)
Manazuru andesite	18	20	70	
Dunham dolomite	53	0	145	Al-Ajmi (2006)
Solenhofen limestone	29	20	80	
Yuubari shale	26	25	50	
dense Marble	35	0	28	
Limestone-Yin	20	0	43.7	Yin et al. (1987)
Sandstone-Yin	20	0	43.7	

Sandstone-Zhang	20	0	4.5	Zhang et al. (1979)
Sandstone-Gao	17	0	9.02	Gao et al. (1993)
Shirahama sandstone	43	5	50	Al-Ajmi (2006)
Maha Sarakham salt	35	0	7	Sriapai et al. (2013)
Soltanieh Granite	57	5	100	Bahrami et al. (2017)
Yamaguchi Marble	27	12.5	40	

2.3 Fitting method

Colmenares and Zoback conducted a systematic study in 2002 using the least squares method to assess the predictive accuracy of six different strength criteria—MC criterion, HB criterion, ML criterion, MWC criterion, exponential Mogi criterion, and DP criterion—based on five sets of true-triaxial strength experimental data. They found that the ML and MWC criteria performed better in predicting the strength of rocks influenced significantly by intermediate principal stress, whereas the MC and HB criteria showed better prediction accuracy for rocks less influenced by intermediate principal stress. It should be noted that at the time of their comparison study, criteria such as MGC criterion and ZZ criterion had not yet been proposed. In this study, based on the least squares method, the objective function for fitting experimental data is formulated as shown in Equation 9. After determining the strength parameters of true-triaxial rock strength experimental data, the fitting performance of different rock strength criteria to the experimental data is evaluated using the squared correlation coefficient of three parameters.

$$R^2 = \left[\frac{N\sum_{i=1}^{N}\sigma_i^m\sigma_i^p - \sum_{i=1}^{N}\sigma_i^m\sum_{i=1}^{N}\sigma_i^p}{\sqrt{\left(N\sum_{i=1}^{N}\left(\sigma_i^m\right)^2 - \left(\sum_{i=1}^{N}\sigma_i^m\right)^2\right)\left(N\sum_{i=1}^{N}\left(\sigma_i^p\right)^2 - \left(\sum_{i=1}^{N}\sigma_i^p\right)^2\right)}} \right]^2 \quad (9)$$

In the equation, σ_i^m, σ_i^p represents the true-triaxial rock strength test value and represents the predicted value, where i denotes the i-th data set, and N denotes the total number of experiments. The evaluation metric R^2, closer to 1, indicates higher fitting accuracy. Using MATLAB numerical analysis software and based on the principles of least squares method, programs were developed to fit five types of rock strength criteria to 32 sets of true-triaxial rock strength experimental data. The research results are presented in the following section.

3. Results and analysis

The calculation results of the square of the correlation coefficient for fitting the HB-type strength criteria are shown in Table 2. For ease of comparison, cumulative frequency distribution plots of the squares of the correlation coefficients are shown in Figures 1 to 5. From the figures, it can be observed that compared to the HB, PH, and GP criteria, the ZZ criterion and HBWW criterion exhibit significant advantages in fitting effectiveness. This is because the HB criterion does not account for the influence of intermediate principal stress, while the PH and GP criteria consider the influence of intermediate principal stress but cannot differentiate the effect of Lode angle on rock strength, leading to larger prediction errors.

Table.2 Squared correlation coefficients for fitting HB-type strength criteria

Rock types	HB	PH	GP	ZZ	HBWW
Aghajari sandstone	0.95	0.77	0.77	0.94	0.98
Asmari Limestone	0.72	0.89	0.89	0.94	0.95
Chaldoran Metapelite	0.90	0.86	0.85	0.94	0.95
Hormoz Salt	0.89	0.94	0.94	0.95	0.95
Inada Granite	0.90	0.94	0.95	0.97	0.97
Jahrom Dolomite	0.80	0.81	0.81	0.95	0.95
Jolfa Marble	0.75	0.92	0.91	0.89	0.84
Karaj Andesite	0.83	0.78	0.79	0.91	0.95
Karaj Trachyte	0.79	0.81	0.81	0.93	0.94
Mahalat Granodiorite	0.87	0.78	0.78	0.89	0.82
Naqade Amphibolite	0.90	0.65	0.65	0.88	0.95
Orikabe Monzonite	0.90	0.94	0.95	0.97	0.97
Pabdeh Shale	0.81	0.86	0.86	0.92	0.91
Shahr-e babak Hornfels	0.73	0.75	0.75	0.90	0.92
shourijeh Siltstone	0.89	0.84	0.84	0.95	0.95
Sandstone-Rukhaiyar	0.83	0.93	0.93	0.94	0.91
Westerly Granite	0.96	0.81	0.82	0.94	0.90
Mizuho Trahchyte	0.91	0.89	0.89	0.95	0.93
KTB amphibolite	0.93	0.78	0.79	0.92	0.96
Manazuru andesite	0.84	0.76	0.76	0.90	0.97
Dunham dolomite	0.86	0.90	0.90	0.97	0.97
Solenhofen limestone	0.74	0.84	0.84	0.93	0.93
Yuubari shale	0.88	0.72	0.72	0.91	0.96
Dense Marble	0.57	0.93	0.92	0.88	0.72
Limestone-Yin	0.93	0.76	0.74	0.92	0.98
Sandstone-Yin	0.96	0.89	0.88	0.97	0.98
Sandstone-Zhang	0.78	0.33	0.33	0.51	0.99
Sandstone-Gao	0.83	0.31	0.31	0.53	0.71

Shirahama sandstone	0.93	0.76	0.76	0.93	0.86
Maha Sarakham salt	0.55	0.72	0.69	0.87	0.76
Soltanieh Granite	0.95	/	0.83	0.94	0.98
Yamaguchi Marble	0.82	0.89	0.89	0.97	0.93

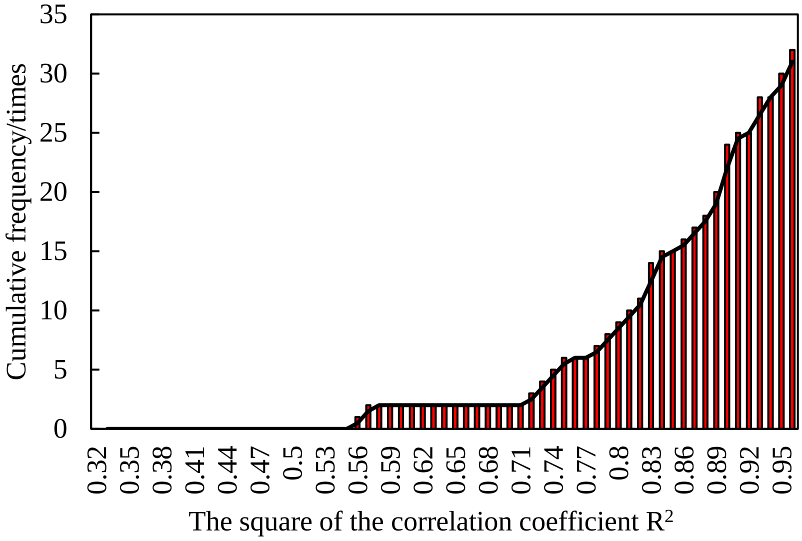

Fig.1 The cumulative frequency distribution plot of the squared correlation coefficients for the HB criterion strength criteria fitting

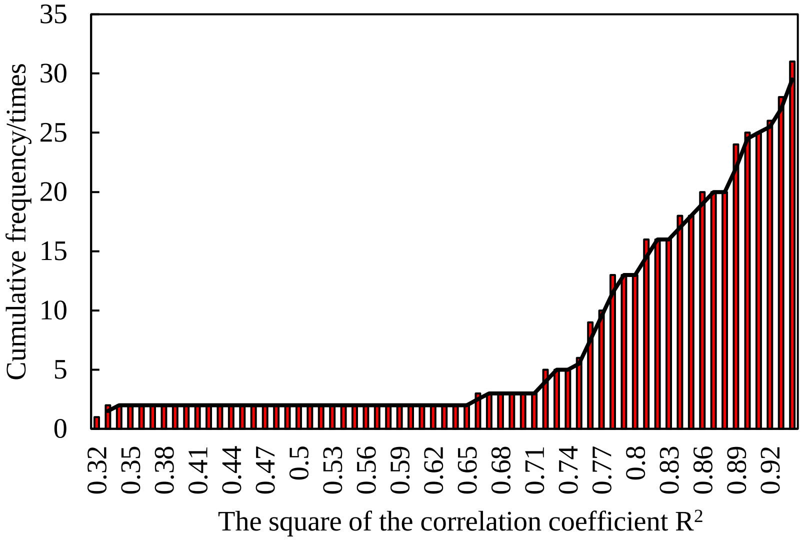

Fig.2 The cumulative frequency distribution plot of the squared correlation coefficients for the PH criterion strength criteria fitting

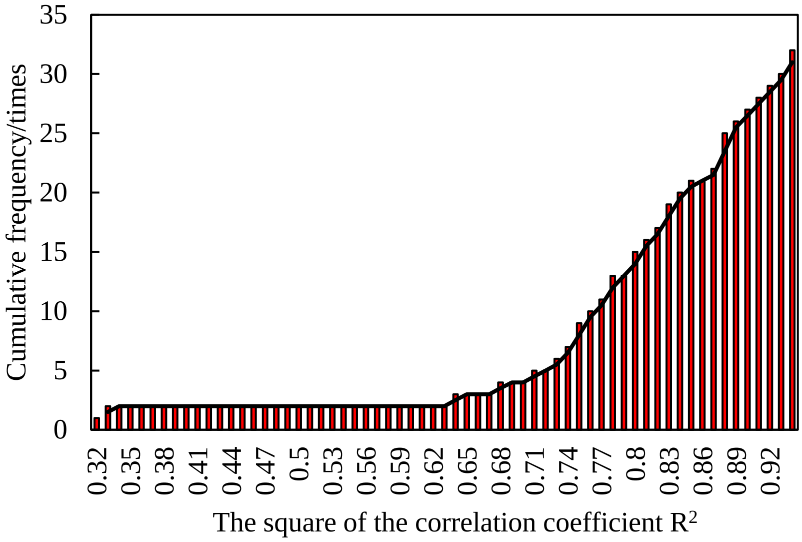

Fig.3 The cumulative frequency distribution plot of the squared correlation coefficients for the GP criterion strength criteria fitting

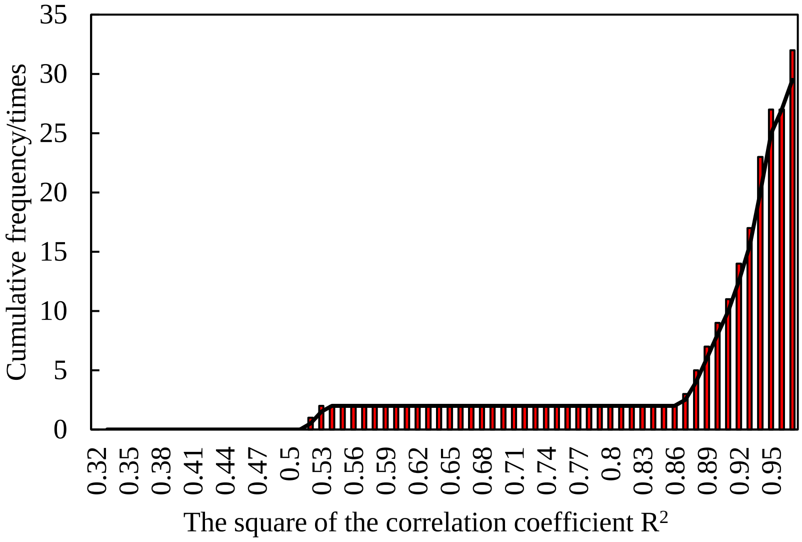

Fig.4 The cumulative frequency distribution plot of the squared correlation coefficients for the ZZ criterion strength criteria fitting

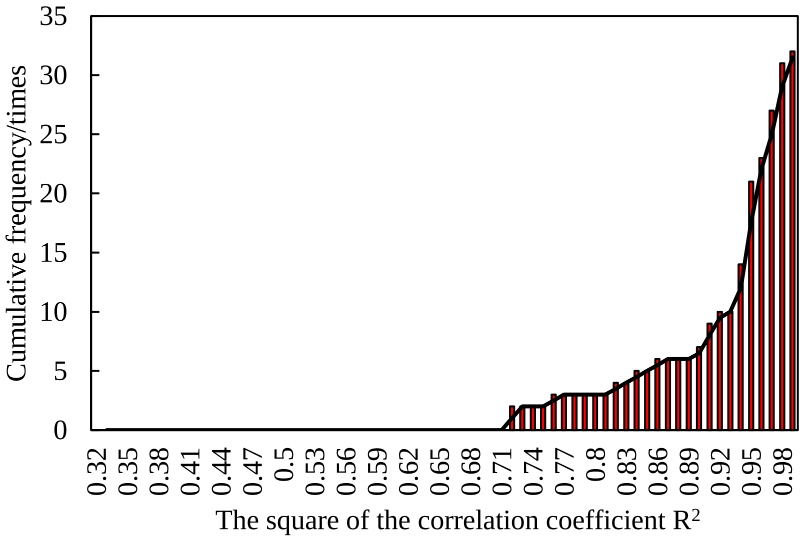

Fig.5 The cumulative frequency distribution plot of the squared correlation coefficients for the HBZZ criterion strength criteria fitting

4. Conclusions

The study primarily concludes the following points, compared to the HB (Hoek-Brown) criterion, PH (Pan-Hudson) criterion, and GP (generalized Priest) criterion, the ZZ (Zhang-Zhu) criterion and HBWW (Hoek-Brown with Willam and Warnke modification) criterion exhibit significant advantages in fitting accuracy. This superiority arises because the HB criterion does not account for the influence of intermediate principal stress, while the PH and GP criteria consider this influence. However, they fail to differentiate the effect of Lode angle on rock strength, leading to larger prediction errors.

References

[1] Yu Maohong. (2007). Unified Theory of Linear and Nonlinear Strength. Chinese Journal of Rock Mechanics and Engineering, 26(4), 662-669.

[2] Zhong Dazhong. (2016). Historical Evolution of Strength Theory for Rock-like Materials (Master's thesis, Nanjing University).

[3] Cai, W., Zhu, H., Liang, W., Zhang, L., & Wu, W. (2021). A New Version of the Generalized Zhang–Zhu Strength Criterion and a Discussion on Its Smoothness and Convexity. Rock Mechanics and Rock Engineering, 1-17.

[4] Yu Maohong. (2014). Advances in strength for materials under complex stress state in the 20th century. Advance in Mechanics, 34(4), 529-560.

[5] Yu, M. H. (2002). Advances in strength theories for materials under complex stress state in the 20th century. Appl. Mech. Rev., 55(3), 169-218.

[6] Jiang, J., & Pietruszczak, S. (1988). Convexity of yield loci for pressure sensitive materials. Computers and Geotechnics, 5(1), 51-63.

[7] Yu Maohong, & Liu Fengyu. (1990). Generalized Bimodal Stress Criterion Corner Model. Acta Mechanica Sinica, 22(2), 213-216.

[8] Shi Shuzhao, & Yang Guanghua. (1987). Improvement of Common Yield Functions for Rock Masses. Chinese Journal of Rock and Soil Mechanics, 9(4), 60-69.

[9] Lee, Y. K., Pietruszczak, S., & Choi, B. H. (2012). Failure criteria for rocks based on smooth approximations to Mohr–Coulomb and Hoek–Brown failure functions. International Journal of Rock Mechanics and Mining Sciences, 56, 146-160.

[10] Drucker, D. C. (1956). On uniqueness in the theory of plasticity. Quarterly of Applied Mathematics, 14(1), 35-42.

[11] Lin, F. B., & Bažant, Z. P. (1986). Convexity of smooth yield surface of frictional material. Journal of engineering mechanics, 112(11), 1259-1262.

[12] Li Bin, Xu Mengguo, & Liu Yanzhang. (2016). Application of Critical State Confining Pressure in the Improvement of Rock Strength Criteria. Rock and Soil Mechanics, 37(2), 390-398.

[13] Zhou Xiaoping, Qian Qihu, & Yang Haiqing. (2008). Strength Criteria for Deep Rock Masses. Chinese Journal of Rock Mechanics and Engineering, 27(1), 117.

[14] Zhou Jijun. (2005). Rock Strength Theory since the 20th Century. In New Advances in Mining Construction Engineering - Proceedings of the 2005 National Mining Construction Academic Conference.

[15] You Mingqing. (2009). Characteristics of Rock Index-type Strength Criteria in Principal Stress Space. Chinese Journal of Rock Mechanics and Engineering, 28(8), 1541-1551.

[16] You Mingqing. (2010). Study on the Mathematical Formulation and Parameter Determination of Rock Strength Criteria. Chinese Journal of Rock Mechanics and Engineering, 29(11), 2172-2184.

[17] You, M. (2011). Comparison of the accuracy of some conventional triaxial strength criteria for intact rock. International journal of rock mechanics and mining sciences (1997), 48(5), 852-863.

[18] Mogi, K. (1967). Effect of the intermediate principal stress on rock failure. Journal of Geophysical Research, 72(20), 5117-5131.

[19] Carter, B. J., Duncan, E. S., & Lajtai, E. Z. (1991). Fitting strength criteria to intact rock. Geotechnical & Geological Engineering, 9(1), 73-81.

[20] You, M. (2010). Mechanical characteristics of the exponential strength criterion under conventional triaxial stresses. International Journal of Rock Mechanics and Mining Sciences, 47(2), 195-204.

[21] Kim, M. K., & Lade, P. V. (1984). Modelling rock strength in three dimensions. In International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 21(1), 21-33. Pergamon.

[22] You, M. (2010). Three independent parameters to describe conventional triaxial compressive strength of intact rocks. Journal of Rock Mechanics and Geotechnical Engineering, 2(4), 350-356.

[23] Zhao, Y. (2021). Retrospection on the development of rock mass mechanics and the summary of some unsolved centennial problems. 40(7): 1297-1336.

[24] Singh, M., Raj, A., & Singh, B. (2011). Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of intact rocks. International Journal of Rock Mechanics and Mining Sciences, 48(4), 546-555.

[25] Singh, M., & Singh, B. (2012). Modified Mohr–Coulomb criterion for non-linear triaxial and poly-axial strength of jointed rocks. International Journal of Rock Mechanics and Mining Sciences, 51, 43-52.

[26] Yang, Q., Zan, Y., & Xie, L. G. (2018). Comparative analysis of the nonlinear unified strength criterion for rocks and other three-dimensional Hoek–Brown strength criteria. Geomechanics and Geophysics for Geo-Energy and Geo-Resources, 4(1), 29-37.

[27] Zan Yuewen, Yu Maohong, & Wang Sijing. (2002). Nonlinear Unified Strength Criterion for Rocks. Chinese Journal of Rock Mechanics and Engineering, 21(10), 1435-1441.

[28] Zan Yuewen, & Yu Maohong. (2013). Generalized Nonlinear Unified Strength Theory for Rocks. Journal of Southwest Jiaotong University, 48(4), 616-624.

[29] Zhang, J., Lin, C., Tang, H., Wen, T., Tannant, D. D., & Zhang, B. (2024). Input-parameter optimization using a SVR based ensemble model to predict landslide displacements in a reservoir area–A comparative study. Applied Soft Computing, 150, 111107.

[30] Lin, J., Dong, C., Lin, C., Duan, D., Ma, P., Zhao, Z., Liu, B., Zhang, X., Huang, X., 2024. The impact of tectonic inversion on diagenesis and reservoir quality of the Oligocene fluvial sandstones: The Upper Huagang Formation, Xihu Depression, East China Sea Shelf Basin. Marine and Petroleum Geology, 106860.

[31] Wang Fengjiao, Xu He, Liu Yikun, et al. Mechanism of Low Chemical Agent Adsorption by High Pressure for Hydraulic Fracturing-Assisted Oil Displacement Technology: A Study of Molecular Dynamics Combined with Laboratory Experiments[J]. Langmuir, 2023, 39(46): 16628-16636.